

Método de Gauss-Jordan

Resuelve los siguientes sistemas de ecuaciones lineales:

$$1) \begin{cases} x - 3z = 4 \\ -x + y - 4z = -4 \\ x + y + z = 345 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & -3 & 4 \\ -1 & -1 & -4 & -4 \\ 1 & 1 & 1 & 345 \end{pmatrix} \xrightarrow[-F_1+F_3]{F_1+F_2} \begin{pmatrix} 1 & 0 & -3 & 4 \\ 0 & 1 & -7 & 0 \\ 0 & 1 & 4 & 341 \end{pmatrix} \xrightarrow{-F_2+F_3} \begin{pmatrix} 1 & 0 & -3 & 4 \\ 0 & 1 & -7 & 0 \\ 0 & 0 & 11 & 341 \end{pmatrix} \xrightarrow{F_3:11} \\ \rightarrow \begin{pmatrix} 1 & 0 & -3 & 4 \\ 0 & 1 & -7 & 0 \\ 0 & 0 & 1 & 31 \end{pmatrix} \xrightarrow[3F_3+F_1]{7F_3+F_2} \begin{pmatrix} 1 & 0 & 0 & 97 \\ 0 & 1 & 0 & 217 \\ 0 & 0 & 1 & 31 \end{pmatrix} \Rightarrow (x, y, z) = (97, 217, 31)$$

$$2) \begin{cases} x + y + z = 106 \\ x - y = -18 \\ x - 4z = -1 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 1 & 106 \\ 1 & -1 & 0 & -18 \\ 1 & 0 & -4 & -1 \end{pmatrix} \xrightarrow[-F_1+F_3]{-F_1+F_2} \begin{pmatrix} 1 & 1 & 1 & 106 \\ 0 & -2 & -1 & -124 \\ 0 & -1 & -5 & -107 \end{pmatrix} \xrightarrow{F_2 \leftrightarrow -F_3} \begin{pmatrix} 1 & 1 & 1 & 106 \\ 0 & 1 & 5 & 107 \\ 0 & -2 & -1 & -124 \end{pmatrix} \rightarrow \\ \xrightarrow{2F_2+F_3} \begin{pmatrix} 1 & 1 & 1 & 106 \\ 0 & 1 & 5 & 107 \\ 0 & 0 & 9 & 90 \end{pmatrix} \xrightarrow{F_3:9} \begin{pmatrix} 1 & 1 & 1 & 106 \\ 0 & 1 & 5 & 107 \\ 0 & 0 & 1 & 10 \end{pmatrix} \xrightarrow[-F_3+F_1]{-5F_3+F_2} \begin{pmatrix} 1 & 1 & 0 & 96 \\ 0 & 1 & 0 & 57 \\ 0 & 0 & 1 & 10 \end{pmatrix} \xrightarrow{-F_2+F_1} \\ \rightarrow \begin{pmatrix} 1 & 0 & 0 & 39 \\ 0 & 1 & 0 & 57 \\ 0 & 0 & 1 & 10 \end{pmatrix} \Rightarrow (x, y, z) = (39, 57, 10)$$

$$3) \begin{cases} x + y + z = 106 \\ x - y = -18 \\ x - 4z = -1 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 1 & 106 \\ 1 & -1 & 0 & -18 \\ 1 & 0 & -4 & -1 \end{pmatrix} \xrightarrow[-F_1+F_3]{-F_2+F_2} \begin{pmatrix} 1 & 1 & 1 & 106 \\ 0 & -2 & -1 & -124 \\ 0 & -1 & -5 & -107 \end{pmatrix} \xrightarrow{F_2 \leftrightarrow (-F_3)} \begin{pmatrix} 1 & 1 & 1 & 106 \\ 0 & 1 & 5 & 107 \\ 0 & -2 & -1 & -124 \end{pmatrix} \xrightarrow{2F_2+F_3} \\ \rightarrow \begin{pmatrix} 1 & 1 & 1 & 106 \\ 0 & 1 & 5 & 107 \\ 0 & 0 & 9 & 90 \end{pmatrix} \xrightarrow{F_3:9} \begin{pmatrix} 1 & 1 & 1 & 106 \\ 0 & 1 & 5 & 107 \\ 0 & 0 & 1 & 10 \end{pmatrix} \xrightarrow[-F_3+F_1]{-5F_3+F_2} \begin{pmatrix} 1 & 1 & 0 & 96 \\ 0 & 1 & 0 & 57 \\ 0 & 0 & 1 & 10 \end{pmatrix} \xrightarrow{-F_2+F_1} \begin{pmatrix} 1 & 0 & 0 & 39 \\ 0 & 1 & 0 & 57 \\ 0 & 0 & 1 & 10 \end{pmatrix} \\ \rightarrow (x, y, z) = (10, 57, 39)$$

$$4) \begin{cases} 2x - y - z = 0 \\ -x + 2y - z = 0 \\ -x + y + 2z = 200 \end{cases}$$

$$\begin{aligned} & \left(\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & 1 & 2 & 200 \end{array} \right) \xrightarrow{F_1 \leftrightarrow -F_2} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \\ -1 & 1 & 2 & 200 \end{array} \right) \xrightarrow{\substack{-2F_1+F_2 \\ F_1+F_3}} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & -1 & 3 & 200 \end{array} \right) \xrightarrow{F_2 \leftrightarrow -F_3} \\ & \rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -200 \\ 0 & 3 & -3 & 0 \end{array} \right) \xrightarrow{-3F_3+F_2} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -200 \\ 0 & 0 & 6 & 600 \end{array} \right) \xrightarrow{F_3:6} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -200 \\ 0 & 0 & 1 & 100 \end{array} \right) \xrightarrow{\substack{3F_3+F_2 \\ -F_3+F_1}} \\ & \rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 0 & -100 \\ 0 & 1 & 0 & 100 \\ 0 & 0 & 1 & 100 \end{array} \right) \xrightarrow{2F_2+F_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 100 \\ 0 & 0 & 1 & 100 \end{array} \right) \Rightarrow (x, y, z) = (100, 100, 100) \end{aligned}$$

$$5) \begin{cases} x + y + z = 50 \\ 2x - y = 1 \\ 2x - 5z = 0 \end{cases}$$

$$\begin{aligned} & \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 2 & -1 & 0 & 1 \\ 2 & 0 & -5 & 0 \end{array} \right) \xrightarrow{\substack{-2F_1+F_2 \\ -2F_1+F_3}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & -3 & -2 & -99 \\ 0 & -2 & -7 & -100 \end{array} \right) \xrightarrow{F_2: (-3)} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 1 & \frac{2}{3} & 33 \\ 0 & -2 & -7 & -100 \end{array} \right) \xrightarrow{2F_2+F_3} \\ & \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 1 & \frac{2}{3} & 33 \\ 0 & 0 & -\frac{17}{3} & -34 \end{array} \right) \xrightarrow{F_3: \left(-\frac{17}{3}\right)} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 1 & \frac{2}{3} & 33 \\ 0 & 0 & 1 & 6 \end{array} \right) \xrightarrow{\substack{-\frac{2}{3}F_3+F_2 \\ -F_3+F_1}} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 44 \\ 0 & 1 & 0 & 29 \\ 0 & 0 & 1 & 6 \end{array} \right) \xrightarrow{-F_2+F_1} \\ & \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 29 \\ 0 & 0 & 1 & 6 \end{array} \right) \Rightarrow (x, y, z) = (15, 29, 6) \end{aligned}$$

$$6) \begin{cases} x - 2y + z = 5 \\ x - 2y = -5 \\ x + y - 27 = -170 \end{cases}$$

$$\begin{aligned} & \left(\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 1 & -2 & 0 & -5 \\ 1 & 1 & -27 & -170 \end{array} \right) \xrightarrow{\substack{-F_1+F_2 \\ -F_1+F_3}} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 0 & -1 & -10 \\ 0 & 3 & -28 & -175 \end{array} \right) \xrightarrow{F_2 \leftrightarrow F_3: 3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 1 & -\frac{28}{3} & -\frac{10}{3} \\ 0 & 0 & -1 & -10 \end{array} \right) \rightarrow \\ & \xrightarrow{F_3: (-1)} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 1 & -\frac{28}{3} & -\frac{10}{3} \\ 0 & 0 & 1 & 10 \end{array} \right) \xrightarrow{\substack{\frac{28}{3}F_3+F_2 \\ -F_3+F_1}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -5 \\ 0 & 1 & 0 & 35 \\ 0 & 0 & 1 & 10 \end{array} \right) \xrightarrow{2F_2+F_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 65 \\ 0 & 1 & 0 & 35 \\ 0 & 0 & 1 & 10 \end{array} \right) \\ & \Rightarrow (x, y, z) = (65, 35, 10) \end{aligned}$$

$$7) \begin{cases} x + y + z = 18400 \\ 3x - 2y = 0 \\ 7y - 5z = 0 \end{cases}$$

$$\begin{aligned} & \left(\begin{array}{ccc|c} 1 & 1 & 1 & 18400 \\ 3 & -2 & 0 & 0 \\ 0 & 7 & -5 & 0 \end{array} \right) \xrightarrow{-3F_1+F_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 18400 \\ 0 & -5 & -3 & -55200 \\ 0 & 7 & -5 & 0 \end{array} \right) \xrightarrow{F_2 \cdot (-5)} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 18400 \\ 0 & 1 & 3/5 & 11040 \\ 0 & 7 & -5 & 0 \end{array} \right) \xrightarrow{-7F_2+F_3} \\ & \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 18400 \\ 0 & 1 & 3/5 & 11040 \\ 0 & 0 & -46/5 & -77280 \end{array} \right) \xrightarrow{F_3 \cdot \left(-\frac{46}{5}\right)} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 18400 \\ 0 & 1 & 3/5 & 11040 \\ 0 & 0 & 1 & 8400 \end{array} \right) \xrightarrow{\begin{matrix} -\frac{3}{5}F_3+F_2 \\ -F_3+F_1 \end{matrix}} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 10000 \\ 0 & 1 & 0 & 6000 \\ 0 & 0 & 1 & 8400 \end{array} \right) \xrightarrow{-F_2+F_1} \\ & \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4000 \\ 0 & 1 & 0 & 6000 \\ 0 & 0 & 1 & 8400 \end{array} \right) \Rightarrow (x, y, z) = (4000, 6000, 8400) \end{aligned}$$

$$8) \begin{cases} 2x + 3y - 4z = 3 \\ -2x - 2y + 3z = -7 \\ 2x - 5y + 2z = -3 \end{cases}$$

$$\begin{aligned} & \left(\begin{array}{ccc|c} 2 & 3 & -4 & 3 \\ -2 & 2 & 3 & -7 \\ 2 & -5 & 2 & -3 \end{array} \right) \xrightarrow{\begin{matrix} F_1+F_2 \\ -F_1+F_3 \end{matrix}} \left(\begin{array}{ccc|c} 2 & 3 & -4 & 3 \\ 0 & 1 & -1 & -4 \\ 0 & -8 & 6 & -6 \end{array} \right) \xrightarrow{\begin{matrix} F_1:2 \\ 8F_2+F_3 \end{matrix}} \left(\begin{array}{ccc|c} 1 & 3/2 & -2 & 3/2 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & -2 & -38 \end{array} \right) \xrightarrow{F_3 \cdot (-2)} \\ & \rightarrow \left(\begin{array}{ccc|c} 1 & 3/2 & -2 & 3/2 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 19 \end{array} \right) \xrightarrow{\begin{matrix} F_3+F_2 \\ 2F_3+F_1 \end{matrix}} \left(\begin{array}{ccc|c} 1 & 3/2 & 0 & 79/2 \\ 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 19 \end{array} \right) \xrightarrow{-\frac{3}{2}F_2+F_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 19 \end{array} \right) \Rightarrow \\ & \Rightarrow (x, y, z) = (17, 15, 19) \end{aligned}$$