

RESOLUCIÓN DE INDETERMINACIONES

1. Calcula los siguientes límites, resolviendo la correspondiente indeterminación:

$$1) \lim_{x \rightarrow +\infty} \frac{6x^2 + 3}{2x^2 - 7x}$$

$$6) \lim_{x \rightarrow 2} \left(\frac{x}{x-2} - \frac{x^2}{x^2-4} \right)$$

$$2) \lim_{x \rightarrow +\infty} \frac{4^x - 2}{4^x + 2}$$

$$7) \lim_{x \rightarrow +\infty} \left[x \left(\sqrt{x^2 + 5} - x \right) \right]$$

$$3) \lim_{x \rightarrow +\infty} \frac{\sqrt{3x^2 - 2x}}{\sqrt{x^2 + 4}}$$

$$8) \lim_{x \rightarrow 0} \left[\left(\frac{2}{\sqrt{x+1}} - 2 \right) \cdot \frac{1}{x} \right]$$

$$4) \lim_{x \rightarrow 2} \frac{x^3 + x^2 - 4x - 4}{x^2 + x - 6}$$

$$9) \lim_{x \rightarrow +\infty} \left(\frac{x+5}{x} \right)^x$$

$$5) \lim_{x \rightarrow 3} \frac{x-3}{\sqrt{6+x}-3}$$

$$10) \lim_{x \rightarrow 0} \frac{1}{2 + 3^{\frac{1}{x}}}$$

Soluciones:

1)

$$\lim_{x \rightarrow +\infty} \frac{6x^2 + 3}{2x^2 - 7x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{\frac{6x^2 + 3}{x^2}}{\frac{2x^2 - 7x}{x^2}} = \lim_{x \rightarrow +\infty} \frac{\frac{6x^2}{x^2} + \frac{3}{x^2}}{\frac{2x^2}{x^2} - \frac{7x}{x^2}} = \lim_{x \rightarrow +\infty} \frac{6 + \frac{3}{x^2}}{2 - \frac{7}{x^2}} = \frac{6}{2} = 3$$

2)

$$\lim_{x \rightarrow +\infty} \frac{4^x - 2}{4^x + 2} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{\frac{4^x - 2}{4^x}}{\frac{4^x + 2}{4^x}} = \lim_{x \rightarrow +\infty} \frac{\frac{4^x}{4^x} - \frac{2}{4^x}}{\frac{4^x}{4^x} + \frac{2}{4^x}} = \lim_{x \rightarrow +\infty} \frac{1 - \frac{2}{4^x}}{1 + \frac{2}{4^x}} = \frac{1 - 0}{1 + 0} = 1$$

3)

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{3x^2 - 2x}}{\sqrt{x^2 + 4}} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \sqrt{\frac{3x^2 - 2x}{x^2 + 4}} = \sqrt{\lim_{x \rightarrow +\infty} \frac{3x^2 - 2x}{x^2 + 4}} = \sqrt{3}$$

4)

$$\lim_{x \rightarrow 2} \frac{x^3 + x^2 - 4x - 4}{x^2 + x - 6} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x^2 + 3x + 2)}{\cancel{(x-2)}(x+3)} = \lim_{x \rightarrow 2} \frac{x^2 + 3x + 2}{x+3} = \frac{2^2 + 3 \cdot 2 + 2}{2+3} = \frac{12}{5}$$

5)

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x-3}{\sqrt{6+x}-3} &= \left[\frac{0}{0} \right] = \lim_{x \rightarrow 3} \frac{(x-3)(\sqrt{6+x}+3)}{(\sqrt{6+x}-3)(\sqrt{6+x}+3)} = \lim_{x \rightarrow 3} \frac{(x-3)(\sqrt{6+x}+3)}{(\sqrt{6+x})^2 - 3^2} = \lim_{x \rightarrow 3} \frac{(x-3)(\sqrt{6+x}+3)}{6+x-9} = \\ &= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(\sqrt{6+x}+3)}{\cancel{x-3}} = \lim_{x \rightarrow 3} (\sqrt{6+x}+3) = \sqrt{6+3}+3 = 6 \end{aligned}$$

6)

$$\lim_{x \rightarrow 2} \left(\frac{x}{x-2} - \frac{x^2}{x^2-4} \right) = [\infty - \infty] \stackrel{(1)}{=} \lim_{x \rightarrow 2} \left(\frac{x(x+2)}{(x-2)(x+2)} - \frac{x^2}{x^2-4} \right) = \lim_{x \rightarrow 2} \frac{x^2+2x-x^2}{x^2-4} = \lim_{x \rightarrow 2} \frac{2x}{x^2-4} \text{ No existe}$$

ya que

$$\lim_{x \rightarrow 2^-} \frac{2x}{x^2-4} = -\infty \text{ y } \lim_{x \rightarrow 2^+} \frac{2x}{x^2-4} = +\infty$$

Donde en (1) hemos tenido en cuenta que:

$$x^2 - 4 = (x+2)(x-2)$$

7)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left[x(\sqrt{x^2+5}-x) \right] &= [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{x(\sqrt{x^2+5}-x)(\sqrt{x^2+5}+x)}{\sqrt{x^2+5}+x} = \lim_{x \rightarrow +\infty} \frac{x(x^2+5-x^2)}{\sqrt{x^2+5}+x} = \\ &= \lim_{x \rightarrow +\infty} \frac{5x}{\sqrt{x^2+5}+x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{\frac{5x}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{5}{x^2}} + \frac{x}{x}} = \lim_{x \rightarrow +\infty} \frac{5}{\sqrt{1 + \frac{5}{x^2}} + 1} = \frac{5}{2} \end{aligned}$$

8)

$$\begin{aligned} \lim_{x \rightarrow 0} \left[\left(\frac{2}{\sqrt{x+1}} - 2 \right) \cdot \frac{1}{x} \right] &= [\infty \cdot 0] = \lim_{x \rightarrow 0} \left[\left(\frac{2-2\sqrt{x+1}}{\sqrt{x+1}} \right) \cdot \frac{1}{x} \right] = \lim_{x \rightarrow 0} \frac{2-2\sqrt{x+1}}{x\sqrt{x+1}} = \left[\frac{\infty}{\infty} \right] = \\ &= \lim_{x \rightarrow 0} \frac{(2-2\sqrt{x+1})(2+2\sqrt{x+1})}{x\sqrt{x+1}(2+2\sqrt{x+1})} = \lim_{x \rightarrow 0} \frac{4-4(x+1)}{x\sqrt{x+1}(2+2\sqrt{x+1})} = \lim_{x \rightarrow 0} \frac{-4\cancel{x}}{\cancel{x}\sqrt{x+1}(2+2\sqrt{x+1})} = \\ &= \frac{-4}{\sqrt{1}(2+2\sqrt{1})} = -1 \end{aligned}$$

9)

Primera forma $\lim_{x \rightarrow +\infty} \left(\frac{x+5}{x} \right)^x = [1^\infty] = e^5$ $\lim_{x \rightarrow +\infty} x \left(\frac{x+5}{x} - 1 \right) = [\infty \cdot 0] = \lim_{x \rightarrow +\infty} x \left(\frac{x+5-x}{x} \right) =$ $= \lim_{x \rightarrow +\infty} \frac{5\cancel{x}}{\cancel{x}} = 5$	Segunda forma $\lim_{x \rightarrow +\infty} \left(\frac{x+5}{x} \right)^x = \lim_{x \rightarrow +\infty} \left(1 + \frac{5}{x} \right)^x =$ $= \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{\frac{x}{5}} \right)^{\frac{x}{5} \cdot 5} = e^5$
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10)

$$\lim_{x \rightarrow 0} \frac{1}{2+3^{\frac{1}{x}}} \Rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} \frac{1}{2+3^{\frac{1}{x}}} = \frac{1}{2+0} = \frac{1}{2} \text{ ya que } 3^{\frac{1}{x}} \xrightarrow{x \rightarrow 0^-} 0 \\ \lim_{x \rightarrow 0^+} \frac{1}{2+3^{\frac{1}{x}}} = 0 \text{ ya que } 3^{\frac{1}{x}} \xrightarrow{x \rightarrow 0^+} +\infty \end{array} \right\} \Rightarrow \nexists \lim_{x \rightarrow 0} \frac{1}{2+3^{\frac{1}{x}}}$$

2. Calcula los siguientes límites:

1) $\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x + 1} \cdot \frac{x^2 + 4}{x^2 - 2x} \right)$

2) $\lim_{x \rightarrow 1} \left(\frac{x + 2}{x - 1} - \frac{x + 5}{x^2 - 1} \right)$

3) $\lim_{x \rightarrow 2} (x - 1)^{\frac{3}{x-2}}$

4) $\lim_{x \rightarrow +\infty} \left(\frac{3x^2 - 5}{3x^2 + x} \right)^{x^2 - 1}$

5) $\lim_{x \rightarrow +\infty} \left(\sqrt{4x^2 - 5} - (2x - 3) \right)$

6) $\lim_{x \rightarrow 2+} \frac{\sqrt{2x - 4}}{x - 2}$

7) $\lim_{x \rightarrow 1-} \frac{2x^2 - 2}{x^2 - 2x + 1}$

8) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x + 8} - 3}$

9) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x}}}{\sqrt{x} + 1}$

10) $\lim_{x \rightarrow 1} \left(\frac{x^2 + 4}{x + 4} \right)^{\frac{x}{x-1}}$

11) $\lim_{x \rightarrow 1} \left(\frac{1 + x}{2 + x} \right)^{\frac{\sqrt{x} - 1}{x-1}}$

12) $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 5} - 3}{\sqrt{x + 7} - 3}$

Soluciones:**1)**

$$\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x + 1} \cdot \frac{x^2 + 4}{x^2 - 2x} \right) = [0 \cdot \infty] = \lim_{x \rightarrow 2} \frac{(x^2 - 4)(x^2 + 4)}{(x + 1)(x^2 - 2x)} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 2} \frac{(x + 2) \cancel{(x - 2)} (x^2 + 4)}{(x + 1)x \cancel{(x - 2)}} = \frac{32}{6} = \frac{16}{3}$$

2)

$$\begin{aligned} \lim_{x \rightarrow 1} \left(\frac{x + 2}{x - 1} - \frac{x + 5}{x^2 - 1} \right) &= [\infty - \infty] = \lim_{x \rightarrow 1} \frac{(x + 2)(x + 1) - (x + 5)}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{(x + 1)(x - 1)} = \lim_{x \rightarrow 1} \frac{\cancel{(x - 1)}(x + 3)}{(x + 1)\cancel{(x - 1)}} = \\ &= \lim_{x \rightarrow 1} \frac{x + 3}{x + 1} = \frac{4}{2} \end{aligned}$$

3)

$$\begin{aligned} \lim_{x \rightarrow 2} (x - 1)^{\frac{3}{x-2}} &= [1^\infty] = e^2 \\ \text{donde } \lim_{x \rightarrow 2} \frac{3}{x - 2} (x - 1 - 1) &= [\infty \cdot 0] = \lim_{x \rightarrow 2} \frac{3 \cancel{(x - 2)}}{\cancel{x - 2}} = \lim_{x \rightarrow 2} 3 = 3 \end{aligned}$$

4)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left(\frac{3x^2 - 5}{3x^2 + x} \right)^{x^2 - 1} &= [1^\infty] = e^{-\infty} = 0 \\ \text{donde } \lim_{x \rightarrow +\infty} (x^2 - 1) \left(\frac{3x^2 - 5}{3x^2 + x} - 1 \right) &= [\infty \cdot 0] = \lim_{x \rightarrow +\infty} (x^2 - 1) \left(\frac{3x^2 - 5}{3x^2 + x} - \frac{3x^2 + x}{3x^2 + x} \right) = \\ &= \lim_{x \rightarrow +\infty} (x^2 - 1) \left(\frac{3x^2 - 5 - (3x^2 + x)}{3x^2 + x} \right) = \lim_{x \rightarrow +\infty} (x^2 - 1) \left(\frac{3x^2 - 5 - 3x^2 - x}{3x^2 + x} \right) = \lim_{x \rightarrow +\infty} (x^2 - 1) \left(\frac{-5 - x}{3x^2 + x} \right) = \end{aligned}$$

$$= \lim_{x \rightarrow +\infty} \frac{(x^2 - 1)(-5 - x)}{3x^2 + x} = \lim_{x \rightarrow +\infty} \frac{-x^3 - 5x^2 + x + 5}{3x^2 + x} = \left[\frac{\infty}{\infty} \right] = -\infty$$

5)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left(\sqrt{4x^2 - 5} - (2x - 3) \right) &= [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(\sqrt{4x^2 - 5} - (2x - 3))(\sqrt{4x^2 - 5} + (2x - 3))}{\sqrt{4x^2 - 5} + (2x - 3)} = \\ &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{4x^2 - 5})^2 - (2x - 3)^2}{\sqrt{4x^2 - 5} + (2x - 3)} = \lim_{x \rightarrow +\infty} \frac{4x^2 - 5 - (4x^2 - 12x + 9)}{\sqrt{4x^2 - 5} + (2x - 3)} = \lim_{x \rightarrow +\infty} \frac{\cancel{4x^2} - 5 - \cancel{4x^2} + 12x - 9}{\sqrt{4x^2 - 5} + (2x - 3)} = \\ &= \lim_{x \rightarrow +\infty} \frac{12x - 14}{\sqrt{4x^2 - 5} + (2x - 3)} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{\frac{12x}{x} - \frac{14}{x}}{\sqrt{\frac{4x^2}{x^2} - \frac{5}{x}} + \frac{2x}{x} - \frac{3}{x}} = \lim_{x \rightarrow +\infty} \frac{12 - \frac{14}{x}}{\sqrt{4 - \frac{5}{x}} + 2 - \frac{3}{x}} = \frac{12}{\sqrt{4} + 2} = \frac{12}{4} = 3 \end{aligned}$$

6)

$$\lim_{x \rightarrow 2+} \frac{\sqrt{2x - 4}}{x - 2} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 2+} \frac{\sqrt{2(x - 2)}}{x - 2} = \lim_{x \rightarrow 2+} \sqrt{\frac{2(x - 2)}{(x - 2)^2}} = \lim_{x \rightarrow 2+} \sqrt{\frac{2}{x - 2}} = +\infty$$

7)

$$\lim_{x \rightarrow 1-} \frac{2x^2 - 2}{x^2 - 2x + 1} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 1-} \frac{2(x^2 - 1)}{x^2 - 2x + 1} = \lim_{x \rightarrow 1-} \frac{2(x + 1)(\cancel{x - 1})}{(x - 1)(\cancel{x - 1})} = \lim_{x \rightarrow 1-} \frac{2(x + 1)}{x - 1} = \left[\frac{4}{0} \right] = -\infty$$

8)

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x + 8} - 3} &= \left[\frac{0}{0} \right] = \lim_{x \rightarrow 1} \frac{(x^2 - 1)(\sqrt{x + 8} + 3)}{(\sqrt{x + 8} - 3)(\sqrt{x + 8} + 3)} = \lim_{x \rightarrow 1} \frac{(x^2 - 1)(\sqrt{x + 8} + 3)}{(\sqrt{x + 8})^2 - 9} = \lim_{x \rightarrow 1} \frac{(x^2 - 1)(\sqrt{x + 8} + 3)}{x + 8 - 9} = \\ &= \lim_{x \rightarrow 1} \frac{(x^2 - 1)(\sqrt{x + 8} + 3)}{x - 1} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 1} \frac{(x + 1)(\cancel{x - 1})(\sqrt{x + 8} + 3)}{\cancel{x - 1}} = \lim_{x \rightarrow 1} (x + 1)(\sqrt{x + 8} + 3) = 12 \end{aligned}$$

9)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x}}}{\sqrt{x} + 1} &= \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x}}(\sqrt{x} + 1)}{(\sqrt{x} + 1)(\sqrt{x} - 1)} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x}}(\sqrt{x} + 1)}{(\sqrt{x})^2 - 1} = \lim_{x \rightarrow +\infty} \frac{\sqrt{(x + \sqrt{x})\sqrt{x}} + \sqrt{x + \sqrt{x}}}{x - 1} = \\ &= \lim_{x \rightarrow +\infty} \frac{\sqrt{x\sqrt{x} + x} + \sqrt{x + \sqrt{x}}}{x - 1} \text{ y no llegamos a ninguna expresión que podamos simplificar...} \end{aligned}$$

Abordamos el problema de otra forma:

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x}}}{\sqrt{x} + 1} = \left[\frac{\infty}{\infty} \right] = \left[\begin{array}{l} \text{hacemos el cambio de variable} \\ u = \sqrt{x} \Rightarrow u^2 = x \\ x \rightarrow +\infty \Rightarrow u \rightarrow +\infty \end{array} \right] = \lim_{u \rightarrow +\infty} \frac{\sqrt{u^2 + u}}{u + 1} = \lim_{u \rightarrow +\infty} \frac{\sqrt{u(u + 1)}}{u + 1} =$$

$$= \lim_{u \rightarrow +\infty} \sqrt{\frac{u(u+1)}{(u+1)^2}} = \lim_{u \rightarrow +\infty} \sqrt{\frac{u}{u+1}} = \sqrt{\lim_{u \rightarrow +\infty} \frac{u}{u+1}} = \sqrt{1} = 1$$

«De otra forma»:

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x+\sqrt{x}}}{\sqrt{x}+1} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{\sqrt{x} \sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{x} \left(1 + \frac{1}{\sqrt{x}} \right)} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1+\frac{1}{\sqrt{x}}}}{1+\frac{1}{\sqrt{x}}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1+\sqrt{\frac{1}{x}}}}{1+\sqrt{\frac{1}{x}}} = \frac{\sqrt{1}}{1} = 1$$

10)

$$\lim_{x \rightarrow 1} \left(\frac{x^2+4}{x+4} \right)^{\frac{x}{x-1}} = [1^\infty] = e^{\frac{1}{5}}$$

$$\begin{aligned} \text{donde } \lim_{x \rightarrow 1} \frac{x}{x-1} \left(\frac{x^2+4}{x+4} - 1 \right) &= \lim_{x \rightarrow 1} \frac{x}{x-1} \left(\frac{x^2+4-x-4}{x+4} \right) = \lim_{x \rightarrow 1} \frac{x}{x-1} \left(\frac{x^2-x}{x+4} \right) = \lim_{x \rightarrow 1} \frac{x}{\cancel{x-1}} \left(\frac{x(\cancel{x-1})}{x+4} \right) = \\ &= \lim_{x \rightarrow 1} \frac{x^2}{x+4} = \frac{1}{5} \end{aligned}$$

11)

$$\lim_{x \rightarrow 1} \left(\frac{1+x}{2+x} \right)^{\frac{\sqrt{x}-1}{x-1}} = \left(\frac{2}{3} \right)^{\frac{1}{2}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{(\sqrt{x})^2-1}{(x-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{\cancel{x-1}}{(\cancel{x-1})(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2}$$

12)

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{\sqrt{x^2+5}-3}{\sqrt{x+7}-3} &= \left[\frac{0}{0} \right] = \lim_{x \rightarrow 2} \frac{(\sqrt{x^2+5}-3)(\sqrt{x+7}+3)}{(\sqrt{x+7}-3)(\sqrt{x+7}+3)} = \lim_{x \rightarrow 2} \frac{(\sqrt{x^2+5}-3)(\sqrt{x+7}+3)}{(\sqrt{x+7})^2-9} = \\ &= \lim_{x \rightarrow 2} \frac{(\sqrt{x^2+5}-3)(\sqrt{x+7}+3)}{x+7-9} = \lim_{x \rightarrow 2} \frac{(\sqrt{x^2+5}-3)(\sqrt{x+7}+3)}{x-2} = \left[\frac{0}{0} \right] = \\ &= \lim_{x \rightarrow 2} \frac{(\sqrt{x^2+5}+3)(\sqrt{x^2+5}-3)(\sqrt{x+7}+3)}{(\sqrt{x^2+5}+3)(x-2)} = \lim_{x \rightarrow 2} \frac{\left[(\sqrt{x^2+5})^2-9 \right](\sqrt{x+7}+3)}{(\sqrt{x^2+5}+3)(x-2)} = \\ &= \lim_{x \rightarrow 2} \frac{[x^2+5-9](\sqrt{x+7}+3)}{(\sqrt{x^2+5}+3)(x-2)} = \lim_{x \rightarrow 2} \frac{(x^2-4)(\sqrt{x+7}+3)}{(\sqrt{x^2+5}+3)(x-2)} = \lim_{x \rightarrow 2} \frac{(x+2)(\cancel{x-2})(\sqrt{x+7}+3)}{(\sqrt{x^2+5}+3)(\cancel{x-2})} = \\ &= \lim_{x \rightarrow 2} \frac{(x+2)(\sqrt{x+7}+3)}{\sqrt{x^2+5}+3} = \frac{24}{6} = 4 \end{aligned}$$

3. Calcula los siguientes límites, resolviendo la correspondiente indeterminación, cuando ésta se presente:

a) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

b) $\lim_{x \rightarrow 3} \frac{2x^2 - 4x - 6}{3 - \sqrt{x + 6}}$

c) $\lim_{x \rightarrow -1} \frac{x^3 + 3x^2 - x - 3}{2x^2 - 2x - 4}$

d) $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^2 - 1}$

e) $\lim_{x \rightarrow 2} \frac{-x^2 - x + 6}{x(x - 2)^2}$

f) $\lim_{x \rightarrow +\infty} \left(\frac{2x^2 + 1}{x^2 + x} \right)^{\sqrt{x^2 + 1} - x}$

g) $\lim_{x \rightarrow 2} \left(\frac{x + 2}{x - 2} - \frac{6x + 4}{x^2 - 4} \right)$

h) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1} - \sqrt{x^2 - 1}}{x}$

i) $\lim_{x \rightarrow 1} \left(\frac{x + 3}{x - 1} \cdot \frac{x^2 + x - 2}{x^2 - 3} \right)$

j) $\lim_{x \rightarrow +\infty} \sqrt{x} (\sqrt{x + 2} - \sqrt{x})$

k) $\lim_{x \rightarrow \frac{1}{2}} \left(\frac{2x + 1}{2x + 2} \right)^{\frac{2x - 1}{\sqrt{2x - 1}}}$

l) $\lim_{x \rightarrow -1} \frac{x + \sqrt{x + 2}}{x + 1}$

m) $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{\sqrt{x} - \sqrt{x}}$

n) $\lim_{x \rightarrow 0} \frac{x}{1 - \sqrt{x + 1}}$

Soluciones:

a)

$$\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 2} \frac{\cancel{(x - 2)} (x^2 + 2x + 4)}{\cancel{x - 2}} = \lim_{x \rightarrow 2} (x^2 + 2x + 4) = 12$$

b)

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{2x^2 - 4x - 6}{3 - \sqrt{x + 6}} &= \left[\frac{0}{0} \right] = \lim_{x \rightarrow 3} \frac{(2x^2 - 4x - 6)(3 + \sqrt{x + 6})}{(3 - \sqrt{x + 6})(3 + \sqrt{x + 6})} = \lim_{x \rightarrow 3} \frac{(2x^2 - 4x - 6)(3 + \sqrt{x + 6})}{9 - (\sqrt{x + 6})^2} = \\ &= \lim_{x \rightarrow 3} \frac{(2x^2 - 4x - 6)(3 + \sqrt{x + 6})}{3 - x} = \lim_{x \rightarrow 3} \frac{2(x + 1)(x - 3)(3 + \sqrt{x + 6})}{3 - x} = \\ &= \lim_{x \rightarrow 3} \frac{2(x + 1)\cancel{(x - 3)}(3 + \sqrt{x + 6})}{-(\cancel{x - 3})} = \lim_{x \rightarrow 3} \left[-2(x + 1)(3 + \sqrt{x + 6}) \right] = -48 \end{aligned}$$

c)

$$\lim_{x \rightarrow -1} \frac{x^3 + 3x^2 - x - 3}{2x^2 - 2x - 4} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow -1} \frac{\cancel{(x + 1)}(x - 1)(x + 3)}{2\cancel{(x + 1)}(x - 2)} = \lim_{x \rightarrow -1} \frac{(x - 1)(x + 3)}{2(x - 2)} = \frac{-4}{-6} = \frac{2}{3}$$

d)

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^2 - 1} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(\sqrt{x} + 1)}{(x^2 - 1)(\sqrt{x} + 1)} = \lim_{x \rightarrow 1} \frac{(\sqrt{x})^2 - 1}{(x^2 - 1)(\sqrt{x} + 1)} = \lim_{x \rightarrow 1} \frac{x - 1}{(x^2 - 1)(\sqrt{x} + 1)} =$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{x-1}}{(x+1)(\cancel{x-1})(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{1}{(x+1)(\sqrt{x}+1)} = \frac{1}{4}$$

e)

$$\lim_{x \rightarrow 2} \frac{-x^2 - x + 6}{x(x-2)^2} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 2} \frac{-(\cancel{x-2})(x+3)}{x(x-2)(\cancel{x-2})} = \lim_{x \rightarrow 2} \frac{-(x+3)}{x(x-2)} = \left[\frac{-5}{0} \right]$$

Límites laterales:

$$\left. \begin{array}{l} \lim_{x \rightarrow 2^-} \frac{-(x+3)}{x(x-2)} = +\infty \\ \lim_{x \rightarrow 2^+} \frac{-(x+3)}{x(x-2)} = -\infty \end{array} \right\} \Rightarrow \nexists \lim_{x \rightarrow 2} \frac{-(x+3)}{x(x-2)} \Rightarrow \nexists \lim_{x \rightarrow 2} \frac{-x^2 - x + 6}{x(x-2)^2}$$

f)

$$\lim_{x \rightarrow +\infty} \left(\frac{2x^2 + 1}{x^2 + x} \right)^{\sqrt{x^2 + 1} - x} = 2^0 = 1$$

$$\begin{aligned} \text{ya que } \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - x) &= [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 1} - x)(\sqrt{x^2 + 1} + x)}{(\sqrt{x^2 + 1} + x)} = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 1})^2 - x^2}{(\sqrt{x^2 + 1} + x)} = \\ &= \lim_{x \rightarrow +\infty} \frac{x^2 + 1 - x^2}{(\sqrt{x^2 + 1} + x)} = \lim_{x \rightarrow +\infty} \frac{1}{(\sqrt{x^2 + 1} + x)} = 0 \end{aligned}$$

g)

$$\begin{aligned} \lim_{x \rightarrow 2} \left(\frac{x+2}{x-2} - \frac{6x+4}{x^2-4} \right) &= [\infty - \infty] = \lim_{x \rightarrow 2} \frac{(x+2)(x+2) - (6x+4)}{x^2-4} = \lim_{x \rightarrow 2} \frac{x(\cancel{x-2})}{(x+2)(\cancel{x-2})} = \\ &= \lim_{x \rightarrow 2} \frac{x}{(x+2)} = \frac{2}{4} = \frac{1}{2} \end{aligned}$$

h)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1} - \sqrt{x^2 - 1}}{x} &= \left[\frac{2}{\infty} \right] = 0 \\ \text{ya que } \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - 1}) &= [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 1} - \sqrt{x^2 - 1})(\sqrt{x^2 + 1} + \sqrt{x^2 - 1})}{\sqrt{x^2 + 1} + \sqrt{x^2 - 1}} = \\ &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 1})^2 - (\sqrt{x^2 - 1})^2}{\sqrt{x^2 + 1} + \sqrt{x^2 - 1}} = \lim_{x \rightarrow +\infty} \frac{x^2 + 1 - x^2 + 1}{\sqrt{x^2 + 1} + \sqrt{x^2 - 1}} = \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{x^2 + 1} + \sqrt{x^2 - 1}} = 0 \end{aligned}$$

i)

$$\begin{aligned}\lim_{x \rightarrow 1} \left(\frac{x+3}{x-1} \cdot \frac{x^2+x-2}{x^2-3} \right) &= [\infty \cdot 0] = \lim_{x \rightarrow 1} \frac{(x+3)(x^2+x-2)}{(x-1)(x^2-3)} = \lim_{x \rightarrow 1} \frac{(x+3) \cancel{(x-1)} (x+2)}{\cancel{(x-1)} (x^2-3)} = \\ &= \lim_{x \rightarrow 1} \frac{(x+3)(x+2)}{x^2-3} = \frac{12}{-2} = -6\end{aligned}$$

j)

$$\begin{aligned}\lim_{x \rightarrow +\infty} \sqrt{x} (\sqrt{x+2} - \sqrt{x}) &= \lim_{x \rightarrow +\infty} \sqrt{x} \cdot \lim_{x \rightarrow +\infty} (\sqrt{x+2} - \sqrt{x}) = [\infty \cdot 0] = \lim_{x \rightarrow +\infty} (\sqrt{x(x+2)} - x) = \\ &= \lim_{x \rightarrow +\infty} (\sqrt{x^2+2x} - x) = [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2+2x} - x)(\sqrt{x^2+2x} + x)}{\sqrt{x^2+2x} + x} = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2+2x})^2 - x^2}{\sqrt{x^2+2x} + x} = \\ &= \lim_{x \rightarrow +\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x} + x} = \lim_{x \rightarrow +\infty} \frac{2x}{\sqrt{x^2+2x} + x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2}} + \frac{x}{x}} = \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{1 + \frac{2}{x}} + 1} = \frac{2}{\sqrt{1} + 1} = 1\end{aligned}$$

ya que $\lim_{x \rightarrow +\infty} (\sqrt{x+2} - \sqrt{x}) = [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x+2} - \sqrt{x})(\sqrt{x+2} + \sqrt{x})}{\sqrt{x+2} + \sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x+2})^2 - (\sqrt{x})^2}{\sqrt{x+2} + \sqrt{x}} =$

$$= \lim_{x \rightarrow +\infty} \frac{x+2-x}{\sqrt{x+2} + \sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{x+2} + \sqrt{x}} = 0$$

k)

$$\begin{aligned}\lim_{x \rightarrow \frac{1}{2}} \left(\frac{2x+1}{2x+2} \right)^{\frac{2x-1}{\sqrt{2x-1}}} &= \left(\frac{2}{3} \right)^0 = 1 \\ \text{donde } \lim_{x \rightarrow \frac{1}{2}} \frac{2x-1}{\sqrt{2x-1}} &= \left[\frac{0}{0} \right] = \lim_{x \rightarrow \frac{1}{2}} \sqrt{\frac{(2x-1)^2}{2x-1}} = \lim_{x \rightarrow \frac{1}{2}} \sqrt{2x-1} = 0\end{aligned}$$

l)

$$\begin{aligned}\lim_{x \rightarrow -1} \frac{x + \sqrt{x+2}}{x+1} &= \left[\frac{0}{0} \right] = \lim_{x \rightarrow -1} \frac{(x + \sqrt{x+2})(x - \sqrt{x+2})}{(x+1)(x - \sqrt{x+2})} = \lim_{x \rightarrow -1} \frac{x^2 - (\sqrt{x+2})^2}{(x+1)(x - \sqrt{x+2})} = \\ &= \lim_{x \rightarrow -1} \frac{x^2 - x - 2}{(x+1)(x - \sqrt{x+2})} = \lim_{x \rightarrow -1} \frac{\cancel{(x+1)}(x-2)}{\cancel{(x+1)}(x - \sqrt{x+2})} = \lim_{x \rightarrow -1} \frac{x-2}{x - \sqrt{x+2}} = \frac{-3}{-1-1} = \frac{3}{2}\end{aligned}$$

m)

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt{x}-\sqrt{x}} &= \left[\frac{0}{0} \right] = \left[\begin{array}{l} \text{hacemos el cambio de variable} \\ u = \sqrt{x} \Rightarrow u^2 = x \\ x \rightarrow 1 \Rightarrow u \rightarrow 1 \end{array} \right] = \lim_{u \rightarrow 1} \frac{u-1}{\sqrt{u^2-u}} = \lim_{u \rightarrow 1} \frac{(u-1)^2}{u(u-1)} = \lim_{u \rightarrow 1} \sqrt{\frac{u-1}{u}} = \\ &= \lim_{u \rightarrow 1} \sqrt{\frac{u-1}{u}} = 0\end{aligned}$$

n)

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{x}{1 - \sqrt{x+1}} &= \left[\frac{0}{0} \right] = \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{x+1})}{(1 - \sqrt{x+1})(1 + \sqrt{x+1})} = \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{x+1})}{1 - (\sqrt{x+1})^2} = \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{x+1})}{1 - x - 1} = \\ &= \lim_{x \rightarrow 0} \frac{\cancel{x}(1 + \sqrt{x+1})}{-\cancel{x}} = \lim_{x \rightarrow 0} \left[-(1 + \sqrt{x+1}) \right] = -2\end{aligned}$$

4. Calcula razonadamente los siguientes límites, resolviendo las correspondientes indeterminaciones, cuando estas se presenten:

a) $\lim_{x \rightarrow +\infty} \left(\frac{x^2 - 1}{x} - \frac{1 + 2x^2}{2x - 1} \right)$

b) $\lim_{x \rightarrow +\infty} (\sqrt{9x^2 + 3x} - 3x)$

c) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 3x} - x)$

Soluciones:

a) $\lim_{x \rightarrow +\infty} \left(\frac{x^2 - 1}{x} - \frac{1 + 2x^2}{2x - 1} \right) = [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(x^2 - 1)(2x - 1) - x(1 + 2x^2)}{x(2x - 1)} =$
 $= \lim_{x \rightarrow +\infty} \frac{\cancel{2x^3} - x^2 - 2x + 1 - x\cancel{2x^3}}{2x^2 - x} = \lim_{x \rightarrow +\infty} \frac{-x^2 - 3x + 1}{2x^2 - x} = -\frac{1}{2}$

b) $\lim_{x \rightarrow +\infty} (\sqrt{9x^2 + 3x} - 3x) = [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(\sqrt{9x^2 + 3x} - 3x)(\sqrt{9x^2 + 3x} + 3x)}{\sqrt{9x^2 + 3x} + 3x} =$
 $= \lim_{x \rightarrow +\infty} \frac{(\sqrt{9x^2 + 3x})^2 - (3x)^2}{\sqrt{9x^2 + 3x} + 3x} = \lim_{x \rightarrow +\infty} \frac{9\cancel{x^2} + 3x - 9\cancel{x^2}}{\sqrt{9x^2 + 3x} + 3x} = \lim_{x \rightarrow +\infty} \frac{3x}{\sqrt{9x^2 + 3x} + 3x} = \left[\frac{\infty}{\infty} \right] =$
 $= \lim_{x \rightarrow +\infty} \frac{\frac{3x}{x}}{\sqrt{9x^2 + \frac{3x}{x}} + \frac{3x}{x}} = \lim_{x \rightarrow +\infty} \frac{3}{\sqrt{9 + \frac{3}{x}} + 3} = \frac{3}{\sqrt{9 + 0} + 3} = \frac{3}{6} = \frac{1}{2}$

c) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 3x} - x) = [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 3x} - x)(\sqrt{x^2 + 3x} + x)}{(\sqrt{x^2 + 3x} + x)} = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 3x})^2 - x^2}{\sqrt{x^2 + 3x} + x} =$
 $= \lim_{x \rightarrow +\infty} \frac{x^2 + 3x - x^2}{\sqrt{x^2 + 3x} + x} = \lim_{x \rightarrow +\infty} \frac{3x}{\sqrt{x^2 + 3x} + x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{3x}{\sqrt{x^2 + \frac{3x}{x}} + \frac{x}{x}} = \lim_{x \rightarrow +\infty} \frac{\frac{3x}{x}}{\sqrt{x^2 + \frac{3}{x}} + \frac{x}{x}} =$
 $= \lim_{x \rightarrow +\infty} \frac{3}{\sqrt{1 + \frac{3}{x}} + 1} = \frac{3}{1 + 1} = \frac{3}{2}$

Resolución de indeterminaciones

INDETERMINACIÓN DEL TIPO $\left[\frac{k}{0} \right]$ CON $k \in (\mathbb{R} - \{0\}) \cup \{\pm\infty\}$

Se calculan los límites laterales: $\lim_{x \rightarrow a+} f(x)$, $\lim_{x \rightarrow a-} f(x)$

Si existen ambos límites y coincide su valor, entonces:

$$\exists \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a+} f(x) = \lim_{x \rightarrow a-} f(x)$$

Si no existe alguno de los límites laterales o no coincide su valor, entonces, no existe $\lim_{x \rightarrow a} f(x)$.

INDETERMINACIÓN DEL TIPO $\left[\frac{0}{0} \right]$

- a) Para funciones racionales: se descomponen numerador y denominador en factores y se simplifica.
- b) Para funciones irracionales: si se trata de una función con raíces cuadradas en el numerador (o en el denominador), multiplicamos numerador y denominador por la expresión conjugada del numerador (o del denominador).

INDETERMINACIÓN DEL TIPO $\left[\frac{\infty}{\infty} \right]$

- a) Para funciones racionales: se divide numerador y denominador por la mayor potencia de x que aparezca en la función (basta con dividir por la mayor potencia de x del denominador).
- b) Para funciones exponenciales: se divide numerador y denominador por la exponencial correspondiente, ya que cuando $x \rightarrow +\infty$, la exponencial crece más rápidamente que cualquier potencia positiva de x .

INDETERMINACIÓN DEL TIPO $[\infty - \infty]$

- a) La función es diferencia de dos funciones racionales: se efectúa dicha operación.
- b) La función es diferencia de funciones irracionales: multiplicamos y dividimos por la expresión conjugada de la función.

INDETERMINACIÓN DEL TIPO $[0 \cdot \infty]$

Transformar esta indeterminación en una de las anteriores, generalmente efectuando las operaciones.

INDETERMINACIÓN DEL TIPO $[1^\infty]$

La indeterminación que nos ocupa se resuelve empleando la siguiente igualdad:

$$\lim_{x \rightarrow a} f(x)^{g(x)} = e^{\lim_{x \rightarrow a} g(x)[f(x)-1]}$$

donde $a \in \mathbb{R} \cup \{\pm\infty\}$.

INDETERMINACIÓN DEL TIPO $[0^0 \text{ o } \infty^0 \text{ o } 0^\infty]$

Estos dos tipos de indeterminaciones se pueden resolver aplicando la siguiente fórmula:

$$\lim_{x \rightarrow a} f(x)^{g(x)} = e^{\lim_{x \rightarrow a} g(x) \log f(x)}$$

donde $a \in \mathbb{R} \cup \{\pm\infty\}$ y $\log = \ln$ es el logaritmo natural.