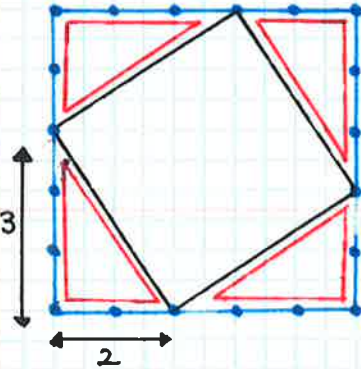


## UNIDAD 10:

# Longitud, área

## 1. INFORMACION



$$A \text{ cuadrado} = e \cdot e = e^2$$

$$A \text{ triángulo} = \frac{b \cdot h}{2}$$

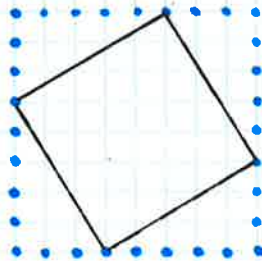


$$A \text{ cuadrado grande} = 5^2 = 25$$

$$A \text{ triángulo} = \frac{2 \cdot 3}{2} = 3$$

$$A \text{ cuadrado coloreado} = \text{cuadrado grande} - 4A \text{ triángulo} = 25 - 4 \cdot 3 = 13$$

## 2. INFORMACION

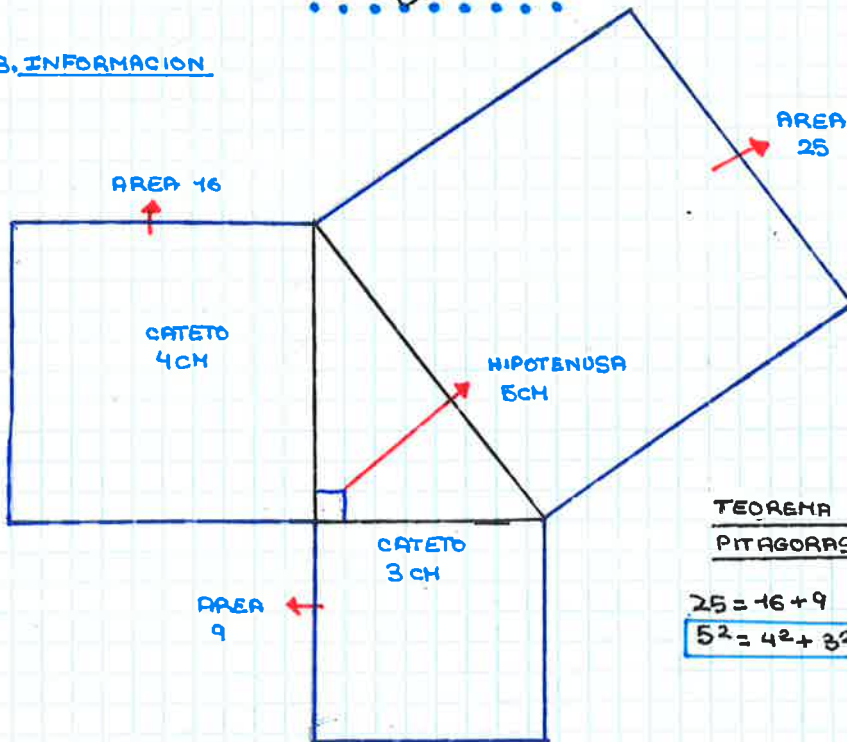


$$A \text{ cuadrado grande: } 8^2 = 64$$

$$A \text{ triángulo: } \frac{5 \cdot 3}{2} = 7,5$$

$$A \text{ cuadrado coloreado: } A \text{ cuadrado grande} - 4A \text{ triángulo} = 64 - 4 \cdot 7,5 = 34$$

## 3. INFORMACION

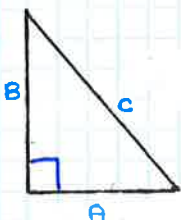


TEOREMA DE  
PITAGORAS

$$25 = 16 + 9$$

$$5^2 = 4^2 + 3^2$$

## TEOREMA DE PITAGORAS



Es un triángulo rectángulo el cuadrado de la hipotenusa es igual a la suma de los cuadrados catetos.

$$c^2 = a^2 + b^2$$





EJERCICIOS

9

$29^2 = 20^2 + d^2$   
 $841 = 400 + d^2$   
 $441 = d^2$   
 $\sqrt{441} = d^2 \rightarrow d = \sqrt{441} = 21 \text{ cm}$

• La distancia es de 21 cm

10

$a^2 = 4^2 + 5^2$   
 $a^2 = 41$   
 $a = \sqrt{41} = 6,40$   
 $b^2 = 12^2 + 5^2$   
 $b^2 = 169$   
 $b = \sqrt{169} = 13$

PERIMETRO

$13 + 13 + 6,40 + 6,40 = 38,8$

• Su perímetro mide 38,8

11

$b^2 = 3^2 + 4^2$   
 $b^2 = 25$   
 $b = \sqrt{25} = 5$

• El lado "b" mide 5 cm y el lado "a"  $6+3=9$  cm

12

$e^2 = 10^2 + 10^2$   
 $e^2 = 200$   
 $e = \sqrt{200} = 14,14$

• Su diagonal mide 14,14 cm.

13

$x^2 = 7,5^2 + 8,5^2$   
 $x^2 = 128,5$   
 $x = \sqrt{128,5} = 11,34$

• Cada lado mide:

EJERCICIOS

14

$20^2 = 12^2 + b^2$   
 $400 = 144 + b^2$   
 $400 - 144 = b^2$   
 $256 = b^2$   
 $\sqrt{256} = b^2 \rightarrow b = \sqrt{256} = 16$

• La diagonal mayor mide  $16 \cdot 2 = \underline{\underline{32 \text{ cm}}}$

15

$x^2 = 3^2 + 8^2$   
 $x^2 = 73$   
 $x = \sqrt{73} = 8,54$

• Cada lado no paralelo mide 8,54 cm



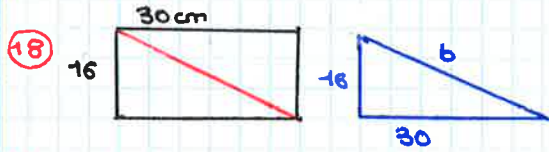
• El otro lado mide 9 cm

$$\begin{aligned}
 15^2 &= 12^2 + x^2 \\
 225 &= 144 + x^2 \\
 225 - 144 &= x^2 \\
 81 &= x^2 \\
 \sqrt{81} &= x^2 \rightarrow x = \sqrt{81} = 9
 \end{aligned}$$



• Cada lado mide 14,14 cm

$$\begin{aligned}
 b^2 &= 10^2 + 10^2 \\
 b^2 &= 200 \\
 b &= \sqrt{200} = 14,14
 \end{aligned}$$



• Su diagonal mide 34 cm

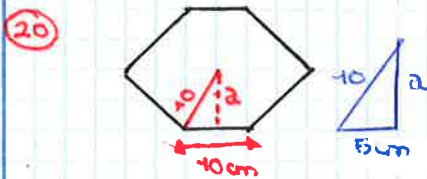
$$\begin{aligned}
 b^2 &= 30^2 + 16^2 \\
 b^2 &= 1156 \\
 b &= \sqrt{1156} = 34
 \end{aligned}$$

2210512023



• La televisión mide 48 pulgadas

$$\begin{aligned}
 d^2 &= 79^2 + 96^2 \\
 d^2 &= 15457 \\
 d &= \sqrt{15457} = 124,32 \text{ cm} = 124,32 : 2,54 \text{ pulgadas} \\
 &= 48,9 \text{ ''}
 \end{aligned}$$



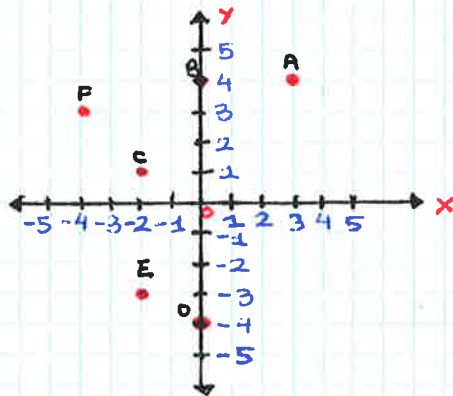
• La apotema mide 8,63 cm

$$\begin{aligned}
 10^2 &= 5^2 + a^2 \\
 100 &= 25 + a^2 \\
 100 - 25 &= a^2 \\
 75 &= a^2 \\
 \sqrt{75} &= a^2 \rightarrow a = \sqrt{75} = 8,63
 \end{aligned}$$

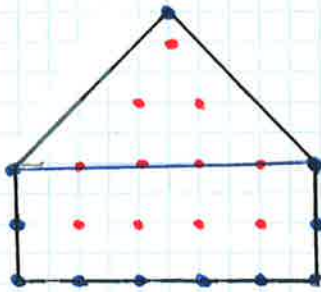
### • INTRODUCCION AL CALCULO DE AREAS

- Coordenadas de un punto en el plano

- A (3,4)
- B (0,4)
- C (-2,1)
- D (0,-4)
- E (-2,-3)
- F (-4,3)



• EJEMPLO 1



1º FORMA: cuando las coordenadas son enteras.

$I = n^\circ$  de puntos en el interior = 10

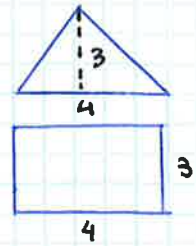
$B = n^\circ$  de puntos en el borde = 10

$$A_{FIGURA} = 10 + \frac{10}{2} - 1 = 14$$

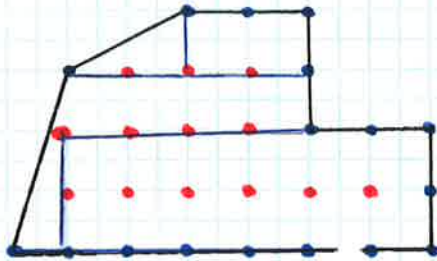
↑ INTERIOR

2º FORMA: mediante formulas

$$A_{FIGURA} = A_{TRIANGULO} + A_{RECTANGULO} = \frac{4 \cdot 3}{2} + 4 \cdot 2 = 14$$



• EJEMPLO 2



1º FORMA: cuando las coordenadas son enteras

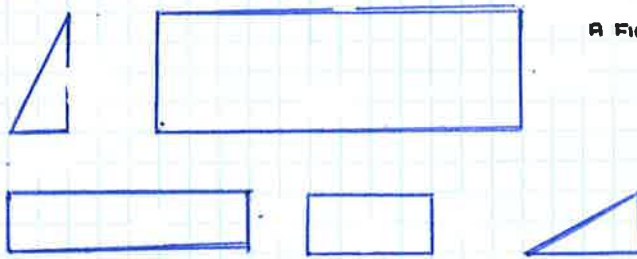
$$A_{FIGURA} = 13 + \frac{17}{2} - 1 = 20,5$$

↑ INTERIOR

$I = 13$   
 $B = 17$

2º FORMA: mediante formulas

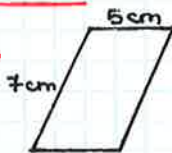
$$A_{FIGURA} = A_{TRIANGULO} + A_{RECTANGULO} =$$



EJERCICIO

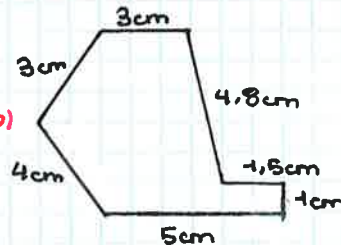
1

a)



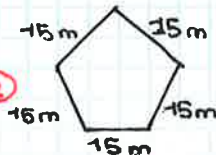
$P = 5 + 7 + 5 + 7 = 24$   
El perimetro de esta figura es de 24 cm.

b)



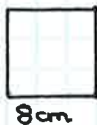
$P = 3 + 3 + 4 + 5 + 1 + 1,5 + 4,8 = 22,3$   
El perimetro de esta figura es de 24 cm.

2

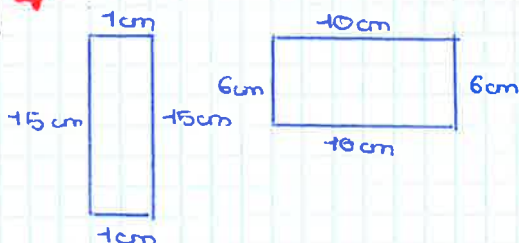


$P = 5 \cdot 15 = 75$   
Necesitamos  $3 \cdot 75 = 225$  m de halambre.

4

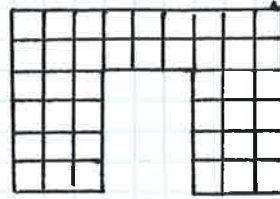
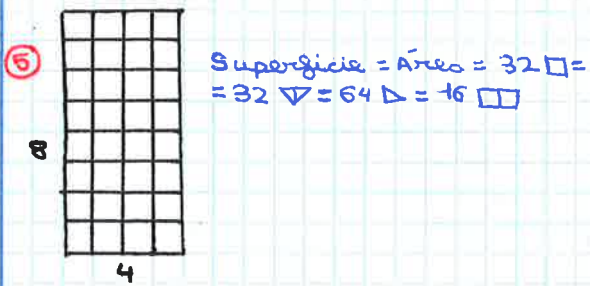


$P_{cuadrado} = 4 \cdot 8 = 32$  cm

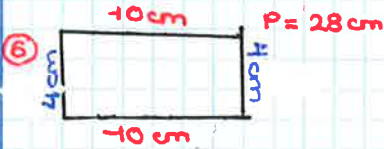


23/05/2023

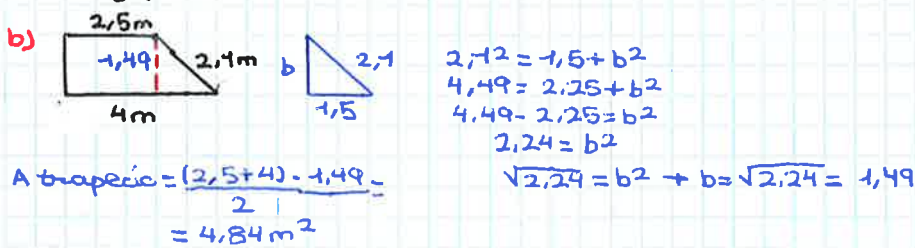
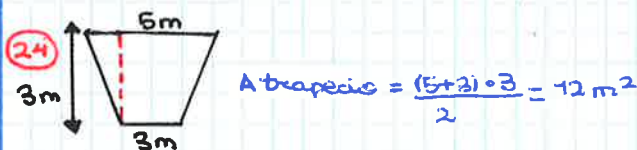
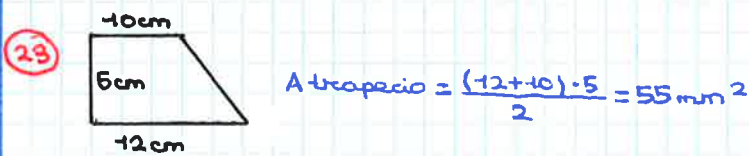
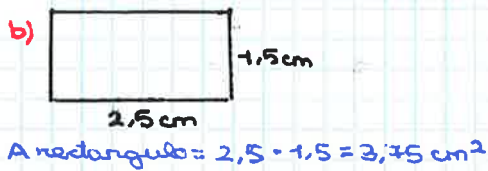
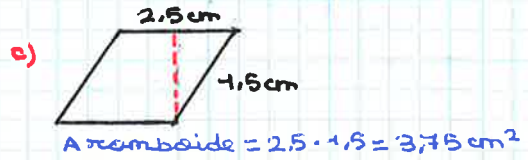
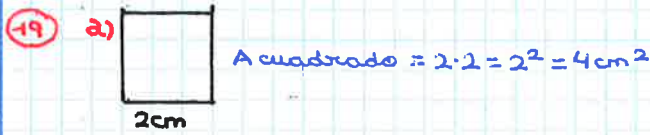




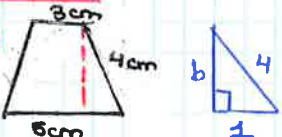
Superficie = área =  $42 \square = 42 \nabla = 84 \triangle = 24 \square$




AREAS DE POLIGONOS



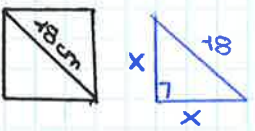
EJERCICIOS

26)   $4^2 = 1^2 + b^2$   
 $16 = 1 + b^2$   
 $16 - 1 = b^2$   
 $12 = b^2 \rightarrow b = \sqrt{12} = 3,46$

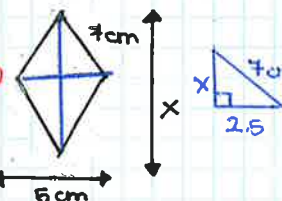
Átrapezoid =  $\frac{(5+3) \cdot 3,46}{2} = 15,48$

27)   $5^2 = 3^2 + x^2$   
 $25 = 9 + x^2$   
 $25 - 9 = x^2$   
 $16 = x^2$   
 $\sqrt{16} = x^2 \rightarrow x = \sqrt{16} = 4$

Átrapezoid =  $\frac{(6+12) \cdot 4}{2} = 36 \text{ m}^2$

28)   $18^2 = x^2 + x^2$   
 $324 = 2x^2$   
 $\frac{324}{2} = x^2$   
 $162 = x^2$   
 $12,73 = \sqrt{162} = x$

Á cuadrado =  $12,73 \cdot 12,73 = 162,05 \text{ cm}^2$

29)   $7^2 = 2,5^2 + x^2$   
 $49 = 6,25 + x^2$   
 $49 - 6,25 = x^2$   
 $42,75 = x^2$   
 $\sqrt{42,75} = x^2 \rightarrow x = \sqrt{42,75} = 6,54$


Á rombo =  $\frac{5 \cdot 12,08}{2} = 12,75 \text{ cm}^2$

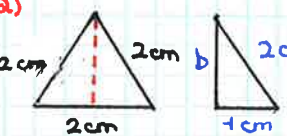
Perímetro =  $7 + 7 + 7 + 7 = 28 \text{ cm}$

26/05/2023

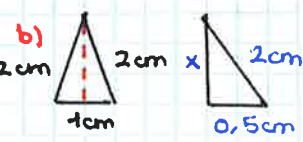
EJERCICIOS

30)   $A \text{ triángulo} = \frac{b \cdot h}{2} = \frac{7 \cdot 4}{2} = 14 \text{ cm}^2$

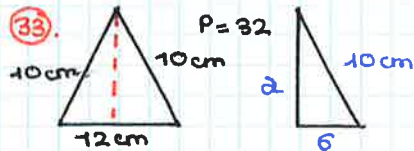
31)   $A \text{ triángulo} = \frac{10 \cdot 6}{2} = 30 \text{ cm}^2$

32) a)   $2^2 = 1^2 + b^2$   
 $4 = 1 + b^2$   
 $4 - 1 = b^2$   
 $3 = b^2$   
 $1,73 = \sqrt{3} = b^2$

Á triángulo =  $\frac{2 \cdot 1,73}{2} = 1,73 \text{ cm}^2$

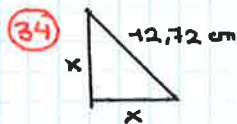
b)   $2^2 = 0,5^2 + x^2$   
 $4 = 0,25 + x^2$   
 $4 - 0,25 = x^2$   
 $3,75 = x^2$   
 $1,94 = \sqrt{3,75} = x^2$

Á triángulo =  $\frac{1 \cdot 1,94}{2} = 0,975 \text{ cm}^2$



$$\begin{aligned}
 10^2 &= 6^2 + a^2 \\
 100 &= 36 + a^2 \\
 100 - 36 &= a^2 \\
 64 &= a^2 \\
 8 &= \sqrt{64} = a
 \end{aligned}$$

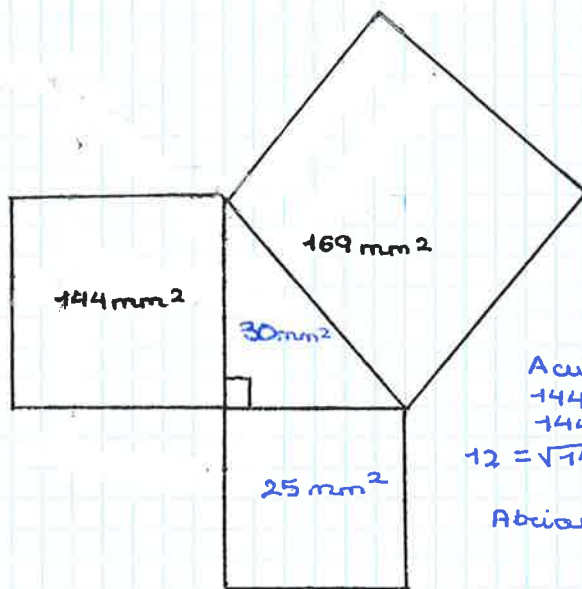
$$A_{\text{triángulo}} = \frac{12 \cdot 8}{2} = 48 \text{ cm}^2$$



$$\begin{aligned}
 12,72 &= x^2 + x^2 \\
 161,80 &= 2x^2 \\
 \frac{161,80}{2} &= x^2 \\
 80,9 &= x^2 \\
 8,99 &= \sqrt{80,9} = x
 \end{aligned}$$

$$\text{Perímetro} = 12,72 + 8,99 + 8,99 = 30,7 \text{ cm}$$

35.



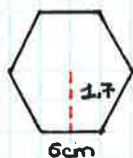
TEOREMA DE PITAGORAS

$$\begin{aligned}
 169 &= 144 + x \\
 169 - 144 &= x \\
 25 &= x
 \end{aligned}$$

$A_{\text{cuadrado}} = e \cdot e$        $A_{\text{cuadrado}} = e \cdot e$   
 $144 = x \cdot x$                $25 = x \cdot x$   
 $144 = x^2$                    $25 = x^2$   
 $12 = \sqrt{144} = x$                $5 = \sqrt{25} = x$   
 $A_{\text{triángulo}} = \frac{12 \cdot 5}{2} = 30 \text{ mm}^2$

29/05/2023

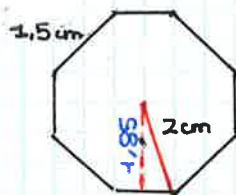
36.



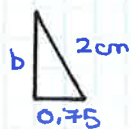
perímetro = 12 cm

$$A_{\text{polígono regular}} = \frac{P \cdot a}{2} = \frac{12 \cdot 1,7}{2} = 10,2 \text{ cm}^2$$

37.



Perímetro = 1,5 · 8 = 12 cm

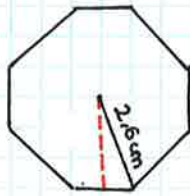


$$\begin{aligned}
 2^2 &= 0,75^2 + b^2 \\
 4 &= 0,56 + b^2 \\
 4 - 0,56 &= b^2 \\
 3,44 &= b^2 \\
 1,85 &= \sqrt{3,44} = b
 \end{aligned}$$

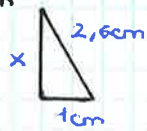
$$A_{\text{polígono regular}} = \frac{12 \cdot 1,85}{2} = 11,1 \text{ cm}^2$$



38



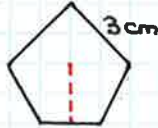
Perímetro  $\therefore 2 \cdot 8 = 16 \text{ cm}$



$$\begin{aligned} 2,6^2 &= 1^2 + x^2 \\ 6,76 &= 1 + x^2 \\ 6,76 - 1 &= x^2 \\ 5,76 &= x^2 \end{aligned}$$

$A_{\text{polígono regular}} = \frac{16 \cdot 2,4}{2} = 19,2 \text{ cm}^2$   $2,4 = \sqrt{5,76} = x^2$

39



$A_{\text{pentágono}} = 15,48 \text{ cm}^2$

$\ell = 3 \text{ cm} \rightarrow p = 3 \cdot 5 = 15 \text{ cm}$   
 $A = 15,48 \text{ cm}^2$   
 ¿apotema?

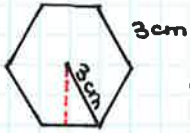
$A = \frac{p \cdot a}{2}$

$15,48 = \frac{15 \cdot a}{2}$

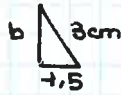
$\frac{30,96}{2} = \frac{15 \cdot a}{2}$

$30,96 = 15 \cdot a$   
 $30,96 - 15 = a$   
 $15,96 = a$

40



Perímetro  $= 3 \cdot 6 = 18 \text{ cm}$



$$\begin{aligned} 3^2 &= 1,5^2 + b^2 \\ 9 &= 2,25 + b^2 \\ 9 - 2,25 &= b^2 \\ 6,75 &= b^2 \\ 2,60 &= \sqrt{6,75} = b^2 \end{aligned}$$

$A_{\text{polígono regular}} = \frac{18 \cdot 2,60}{2} = 23,4 \text{ cm}^2$

EJERCICIOS

1/06/2023

- 42) a)  $A = \pi \cdot r^2 = \pi \cdot 50^2 = 7853,98 \text{ cm}^2$
- b)  $A = \pi \cdot r^2 = \pi \cdot 7^2 = 153,94 \text{ km}^2$
- c)  $A = \pi \cdot r^2 = \pi \cdot 0,25^2 = 0,20 \text{ m}^2$

- 43) a)  $A = \pi \cdot 5^2 \cdot \frac{48}{360} = 10,47 \text{ cm}^2$
- b)  $A = \pi \cdot 3^2 = 28,27 \text{ cm}^2$   
 $A = \pi \cdot 2^2 = 12,57 \text{ cm}^2 \rightarrow 28,27 - 12,57 = 15,7 \text{ cm}^2$

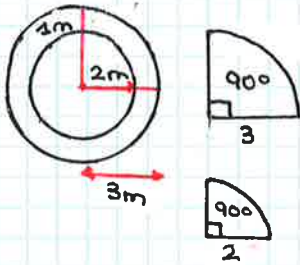
44)  $A = \frac{\pi \cdot 2^2 \cdot 12}{360} = 0,42 \text{ dm}^2$

45)  $A = \pi \cdot 6^2 - \pi \cdot 8^2 = 87,96 \text{ mm}^2$

46) a)  $A = \frac{\pi \cdot 5^2 \cdot 54}{360} \cdot 6 = 35,34 \text{ cm}^2$

b)  $A = \frac{\pi \cdot 5^2 \cdot 35}{360} - \frac{\pi \cdot 2^2 \cdot 35}{360} = 6,41 \text{ cm}^2$

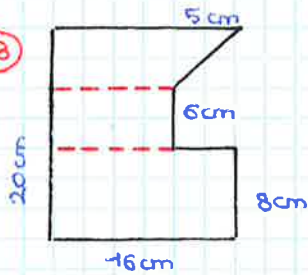
47



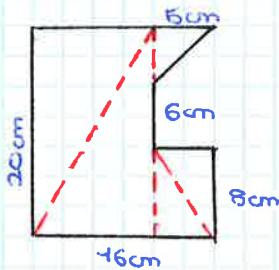
$$A = \frac{\pi \cdot 3^2 \cdot 90}{360} - \frac{\pi \cdot 2^2 \cdot 90}{360} = 3,93 \text{ m}^2$$

2/06/2023

48

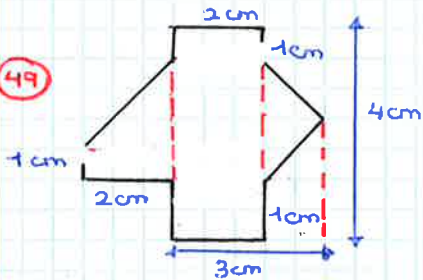


$$\begin{aligned} A_{\text{figura}} &= A_{\text{rectangulo grande}} + A_{\text{rectangulo pequeno}} + A_{\text{trapecio}} \\ &= 16 \cdot 8 + 11 \cdot 6 + \frac{(16+11) \cdot 6}{2} = 275 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} A_{\text{figura}} &= 2 \cdot A_{\text{triangulo}} + 2 \cdot A_{\text{triangulo}} + A_{\text{triangulo}} = \\ &= 2 \cdot \frac{11 \cdot 20}{2} + 2 \cdot \frac{5 \cdot 8}{2} + \frac{5 \cdot 6}{2} = 275 \text{ cm}^2 \end{aligned}$$

49



$$\begin{aligned} A_{\text{figura}} &= A_{\text{rectangulo}} + A_{\text{triangulo}} + A_{\text{trapecio}} = \\ &= 2 \cdot 4 + \frac{2 \cdot 1}{2} + \frac{(2+1) \cdot 2}{2} = 12 \text{ cm}^2 \end{aligned}$$

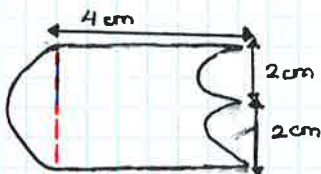
50



$$\begin{aligned} A_{\text{figura}} &= A_{\text{cuadrado}} + A_{\text{triangulo grande}} + A_{\text{triangulo pequeno}} \\ A &= 4 \cdot 4 + \frac{4 \cdot 2}{2} + \frac{4 \cdot 1}{2} = 22 \text{ cm}^2 \\ \text{Perimetro} &= 4 + 4 + 2 \cdot 2,83 + 2 \cdot 2,24 = 18,14 \text{ cm} \end{aligned}$$

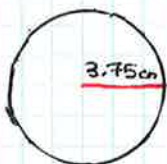
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51

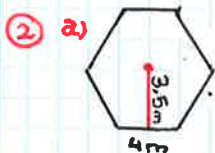


$$\begin{aligned} A_{\text{figura}} &= A_{\text{semicirculo}} + A_{\text{cuadrado}} - A_{\text{circulo}} \\ &= \frac{\pi \cdot 2^2}{2} + 4 \cdot 4 - \pi \cdot 2^2 = 49,14 \text{ cm}^2 \end{aligned}$$

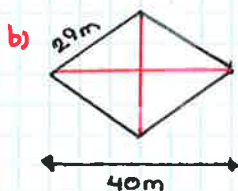
55



$$A_{\text{circulo}} = \pi \cdot 3,75^2 = 44,18 \text{ cm}^2$$



$$A_{\text{hexagono}} = \frac{24 \cdot 3,5}{2} = 42 \text{ m}^2$$



$$29^2 = 20^2 + b^2$$

$$841 = 400 + b^2$$

$$841 - 400 = b^2$$

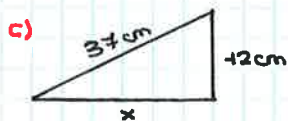
$$441 = b^2$$

$$21 = \sqrt{441} = b$$

La otra diagonal mide

$$2 \cdot 21 = 42 \text{ m}$$

$$A_{\text{rombo}} = \frac{42 \cdot 40}{2} = 840 \text{ m}^2$$



$$37^2 = 12^2 + x^2$$

$$1369 = 144 + x^2$$

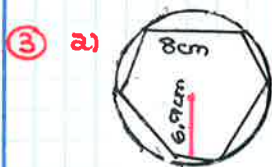
$$1369 - 144 = x^2$$

$$1225 = x^2$$

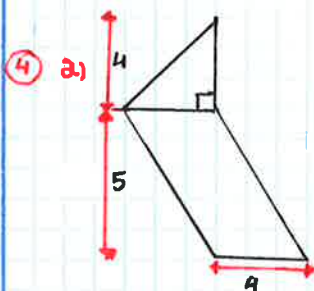
$$35 = \sqrt{1225} = x$$

$$A_{\text{triangulo}} = \frac{35 \cdot 12}{2} = 210 \text{ cm}^2$$

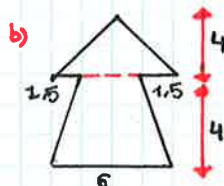
d)  $A_{\text{poligono regular}} = \frac{42 \cdot 5,8}{2} = 121,8 \text{ cm}^2$



$$A_{\text{figura}} = A_{\text{circulo}} - A_{\text{hexagono}} \\ = \pi \cdot 8^2 - \frac{48 \cdot 6,9}{2} = 35,46 \text{ cm}^2$$

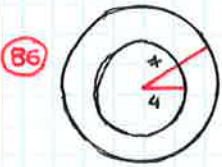


$$A_{\text{figura}} = A_{\text{triangulo}} + A_{\text{trapezoide}} \\ = \frac{4 \cdot 4}{2} + 4 \cdot 5 = 28 \text{ cm}^2$$

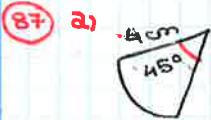


$$A_{\text{figura}} = A_{\text{triangulo}} + A_{\text{trapezio}} \\ = \frac{6 \cdot 4}{2} + \frac{(6+3) \cdot 4}{2} = 30 \text{ cm}^2$$

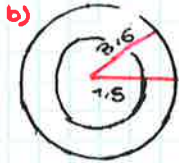




A colorada = A círculo grande - A círculo pequeno =  
 $= \pi \cdot 7^2 - \pi \cdot 4^2 = 103,67 \text{ cm}^2$



A sector circular =  $\frac{\pi \cdot 4^2 \cdot 45}{360} = 6,28 \text{ cm}^2$



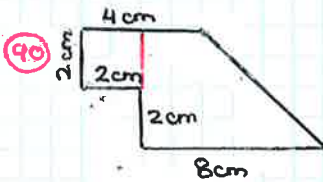
A colorada = A círculo grande - A círculo pequeno  
 $= \pi \cdot 3,6^2 - \pi \cdot 1,5^2 = 33,65 \text{ cm}^2$



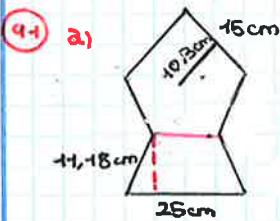
48,57 cm<sup>2</sup>

$48,57 = \frac{\pi \cdot 78^2 \cdot n^\circ}{360}$

$48,57 \cdot 360 = \pi \cdot 78^2 \cdot n^\circ$   
 $= \frac{48,57 \cdot 360}{\pi \cdot 78^2} = n^\circ$



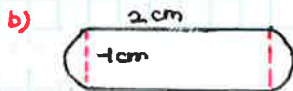
A figura = A quadrado + A trapézio  
 $= 2 \cdot 2 + \frac{(8+2) \cdot 4}{2} = 24 \text{ cm}^2$



A figura = A pentágono + A trapézio  
 $= \frac{75 \cdot 10,3}{2} + \frac{(25+15) \cdot 10}{2} = 586,25 \text{ cm}^2$



$11,18^2 = 5^2 + x^2$   
 $124,99 = 25 + x^2$   
 $124,99 - 25 = x^2$   
 $99,99 = x^2$   
 $10 \text{ cm} = \sqrt{99,99} = x$

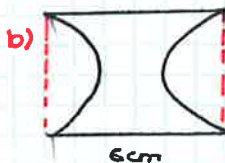


A figura = A retângulo + A círculo  
 $= 1 \cdot 2 + \pi \cdot 0,5^2 = 2,79 \text{ cm}^2$

09/06/2023



A figura =  $\frac{\pi \cdot 4^2 - \pi \cdot 5^2}{2} = 87,96 \text{ cm}^2$



A figura = A retângulo - A círculo  
 $= 6 \cdot 4 - \pi \cdot 2^2 = 11,43 \text{ cm}^2$