

Calcula las siguientes integrales:

1)  $\int \frac{3x-2}{x^2-2x+1} dx$  (ejercicio 563)

Descomponemos en fracciones simples:

$$x^2 - 2x + 1 = 0 \Rightarrow x = 1 \text{ (doble)}$$

$$\frac{3x-2}{x^2-2x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{A(x-1)+B}{(x-1)^2} \Rightarrow 3x-2 = A(x-1)+B \Rightarrow \begin{cases} A=3 \\ -2 = -A+B \end{cases} \Rightarrow (A,B) = (3,1)$$

$$\frac{3x-2}{x^2-2x+1} = \frac{3}{x-1} + \frac{1}{(x-1)^2}$$

Integramos:

$$\int \frac{3x-2}{x^2-2x+1} dx = 3 \int \frac{1}{x-1} dx + \int \frac{1}{(x-1)^2} dx = 3 \log|x-1| + \int (x-1)^{-2} dx = 3 \log|x-1| - \frac{1}{x-1} + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

2)  $\int_0^1 (x+1)e^{-x} dx$  (ejercicio 564)

Calculamos una primitiva de  $(x+1)e^{-x}$ :

$$\int (x+1)e^{-x} dx = \left[ \begin{array}{l} u = x+1 \xrightarrow{\text{derivamos}} du = dx \\ dv = e^{-x} dx \xrightarrow{\text{integramos}} v = -e^{-x} \end{array} \right] = -(x+1)e^{-x} + \int e^{-x} dx = -(x+1)e^{-x} - e^{-x} = (-x-2)e^{-x} + C = G(x)$$

Aplicamos la regla de Barrow:

$$\int_0^1 (x+1)e^{-x} dx = G(1) - G(0) = -3e^{-1} - (-2) = 2 - \frac{3}{e}$$

3)  $\int \frac{1}{\sqrt{x}(x+1)} dx$  (ejercicio 564)

$$\int \frac{1}{\sqrt{x}(x+1)} dx = \left[ \begin{array}{l} t = \sqrt{x} \xrightarrow{\text{derivando}} dt = \frac{1}{2\sqrt{x}} dx \\ t^2 = x \end{array} \right] = 2 \int \frac{1}{1+t^2} dt = 2 \arctg t = 2 \arctg(\sqrt{x}) + C$$

4)  $\int \frac{x^3+2x^2+x-10}{x^2+x-2} dx$  (ejercicio 567)

Efectuamos la división:

$$\begin{array}{r} x^3 + 2x^2 + x - 10 \quad | \quad x^2 + x - 2 \\ -x^3 - x^2 + 2x \quad \quad \quad x+1 \\ \hline x^2 + 3x - 10 \\ -x^2 - x + 2 \\ \hline 2x - 8 \end{array} \qquad \frac{x^3+2x^2+x-10}{x^2+x-2} = x+1 + \frac{2x-8}{x^2+x-2}$$

Descomponemos en fracciones simples:

$$x^2 + x - 2 = (x-1)(x+2)$$

$$\frac{2x-8}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{(x-1)(x+2)} \Rightarrow$$

$$\Rightarrow 2x-8 = A(x+2)+B(x-1) \Rightarrow \begin{cases} \text{Si } x=1 \Rightarrow -6=3A \Rightarrow A=-2 \\ \text{Si } x=-2 \Rightarrow -12=-3B \Rightarrow B=4 \end{cases}$$

$$\frac{2x-8}{x^2+x-2} = \frac{-2}{x-1} + \frac{4}{x+2}$$

Integramos:

$$\int \frac{x^3 + 2x^2 + x - 10}{x^2 + x - 2} dx = \int (x+1) dx + \int \frac{-2}{x-1} dx + \int \frac{4}{x+2} dx =$$

$$= \frac{x^2}{2} + x - 2 \log|x-1| + 4 \log|x+2| + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

5)  $\int x^2 \log x dx$  (ejercicio 567)

$$\int x^2 \log x dx = \left[ \begin{array}{l} u = \log x \xrightarrow{\text{derivamos}} du = \frac{1}{x} dx \\ dv = x^2 dx \xrightarrow{\text{integramos}} v = \frac{x^3}{3} \end{array} \right] = \frac{1}{3} x^3 \log x - \int \frac{x^3}{3} \frac{1}{x} dx = \frac{1}{3} x^3 \log x - \frac{1}{3} \int x^2 dx =$$

$$= \frac{1}{3} x^3 \log x - \frac{1}{3} \frac{x^3}{3} = \frac{1}{3} x^3 \log x - \frac{x^3}{9} + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

6)  $\int \frac{2 \operatorname{sen} x \cos x}{1 + \operatorname{sen}^2 x} dx$  (ejercicio 570)

$$\int \frac{2 \operatorname{sen} x \cos x}{1 + \operatorname{sen}^2 x} dx = \left[ \begin{array}{l} t = 1 + \operatorname{sen}^2 x \\ dt = 2 \operatorname{sen} x \cos x dx \end{array} \right] = \int \frac{1}{t} dt = \log|t| = \log|1 + \operatorname{sen}^2 x + C|$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

7)  $\int \frac{x^2 + x - 4}{x^3 - 4x} dx$  (ejercicio 570)

Descomponemos en fracciones simples:

$$x^3 - 4x = x(x^2 - 4) = x(x+2)(x-2)$$

$$\frac{x^2 + x - 4}{x^3 - 4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} = \frac{A(x-2)(x+2) + Bx(x+2) + Cx(x-2)}{x(x-2)(x+2)} \Rightarrow$$

$$\Rightarrow A(x-2)(x+2) + Bx(x+2) + Cx(x-2) = x^2 + x - 4 \Rightarrow$$

$$\Rightarrow \begin{cases} x=2 \Rightarrow A(2-2)(2+2)+2B(2+2)+2C(2-2)=2^2+2-4 \Rightarrow B=\frac{1}{4} \\ x=0 \Rightarrow A(0-2)(0+2)+B \cdot 0 \cdot (0+2)+C \cdot 0 \cdot (0-2)=0^2+0-4 \Rightarrow A=1 \\ x=-2 \Rightarrow C=-\frac{1}{4} \end{cases}$$

$$\frac{x^2+x-4}{x^3-4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} = \frac{1}{x} + \frac{\frac{1}{4}}{x-2} + \frac{-\frac{1}{4}}{x+2}$$

Integramos:

$$\int \frac{x^2+x-4}{x^3-4x} dx = \int \frac{1}{x} dx + \frac{1}{4} \int \frac{1}{x-2} dx - \frac{1}{4} \int \frac{1}{x+2} dx = \log|x| + \frac{1}{4} \log|x-2| - \frac{1}{4} \log|x+2| + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

8)  $\int \frac{2 \cos x}{1 + \sin^2 x} dx$  (ejercicio 572)

$$\int \frac{2 \cos x}{1 + \sin^2 x} dx = \left[ \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \right] = 2 \int \frac{1}{1+t^2} dt = 2 \arctg t = 2 \arctg(\sin x) + C$$

9)  $\int (x^2 + 2x) \log x dx$  (ejercicio 572)

$$\int (x^2 + 2x) \log x dx = \left[ \begin{array}{l} u = \log x \xrightarrow{\text{derivamos}} du = \frac{1}{x} dx \\ dv = (x^2 + 2x) \xrightarrow{\text{integramos}} v = \frac{x^3}{3} + x^2 \end{array} \right] = \left( \frac{x^3}{3} + x^2 \right) \log x - \int \left( \frac{x^3}{3} + x^2 \right) \frac{1}{x} dx =$$

$$= \left( \frac{x^3}{3} + x^2 \right) \log x - \int \left( \frac{x^2}{3} + x \right) dx = \left( \frac{x^3}{3} + x^2 \right) \log x - \frac{x^3}{9} - \frac{x^2}{2} + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

10)  $\int \sin^2 x \cos x dx$  (ejercicio 575)

Es inmediata de tipo potencial:  $\int \sin^2 x \cos x dx = \frac{\sin^3 x}{3} + C$

También se puede calcular mediante un cambio de variable:

$$\int \sin^2 x \cos x dx = \left[ \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \right] = \int t^2 dt = \frac{t^3}{3} = \frac{\sin^3 x}{3} + C$$

11)  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$  (ejercicio 575)

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \left[ \begin{array}{l} t = \sqrt{x} \\ dt = \frac{1}{2\sqrt{x}} dx \end{array} \right] = \int 2e^t dt = 2e^t = 2e^{\sqrt{x}} + C$$

**12)**  $\int \frac{x^2 - 3x + 1}{x^3 - 5x^2 + 8x - 4} dx$

Descomponemos en fracciones simples:

$$x^3 - 5x^2 + 8x - 4 = (x-1)(x-2)^2 \quad (\text{aplicando la regla de Ruffini})$$

$$\frac{x^2 - 3x + 1}{x^3 - 5x^2 + 8x - 4} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2} = \frac{A(x-2)^2 + B(x-1)(x-2) + C(x-1)}{(x-1)(x-2)^2} \Rightarrow$$

$$\Rightarrow x^2 - 3x + 1 = A(x-2)^2 + B(x-1)(x-2) + C(x-1) \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \text{Si } x=1 \Rightarrow A=-1 \\ \text{Si } x=2 \Rightarrow C=-1 \\ \text{Derivamos dos veces: } x^2 - 3x + 1 = -(x-2)^2 + B(x-1)(x-2) - (x-1) \Rightarrow \\ 2x - 3 = -2(x-2) + 2Bx - 1 \\ 2 = -2 + 2B \Rightarrow B=2 \end{array} \right.$$

$$\Rightarrow \frac{x^2 - 3x + 1}{x^3 - 5x^2 + 8x - 4} = \frac{-1}{x-1} + \frac{2}{x-2} + \frac{-1}{(x-2)^2}$$

Integramos:

$$\int \frac{x^2 - 3x + 1}{x^3 - 5x^2 + 8x - 4} dx = \int \frac{-1}{x-1} dx + \int \frac{2}{x-2} dx + \int \frac{-1}{(x-2)^2} dx = -\log|x-1| + 2\log|x-2| + \frac{1}{x-2} + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

**13)**  $\int x \log x dx$  (ejercicio 580)

$$\int x \log x dx = \left[ \begin{array}{l} u = \log x \xrightarrow{\text{derivamos}} du = \frac{1}{x} dx \\ dv = x dx \xrightarrow{\text{integramos}} v = \frac{x^2}{2} \end{array} \right] = \frac{1}{2} x^2 \log x - \int \frac{x^2}{2} \frac{1}{x} dx = \frac{1}{2} x^2 \log x - \frac{1}{2} \int x dx =$$

$$= \frac{1}{2} x^2 \log x - \frac{1}{2} \frac{x^2}{2} = \frac{1}{2} x^2 \log x - \frac{1}{4} x^2 + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

**14)**  $\int \frac{\sqrt{x}}{1+\sqrt{x}} dx$  (ejercicio 580)

$$\int \frac{\sqrt{x}}{1+\sqrt{x}} dx = \left[ \begin{array}{l} t = \sqrt{x} \Rightarrow t^2 = x \\ dx = 2t dt \end{array} \right] = \int \frac{t}{1+t} 2t dt = 2 \int \frac{t^2}{1+t} dt$$

Descomponemos en fracciones simples  $\frac{t^2}{1+t}$ :

$$\frac{t^2}{1+t} = \frac{-t^2 - t}{-t} + \frac{|t+1|}{t-1} \Rightarrow \frac{t^2}{t-1} = t-1 + \frac{1}{t+1}$$

$$\frac{t^2}{t-1} = t-1 + \frac{1}{t+1}$$

Integramos:

$$\int \frac{t^2}{t-1} dt = \int (t-1) dt + \int \frac{1}{t+1} dt = \frac{t^2}{2} - t + \log|t+1|$$

$$\int \frac{\sqrt{x}}{1+\sqrt{x}} dx = \left[ \begin{array}{l} t = \sqrt{x} \Rightarrow t^2 = x \\ dx = 2t dt \end{array} \right] = \int \frac{t}{1+t} 2t dt = 2 \int \frac{t^2}{1+t} dt = 2 \left( \frac{t^2}{2} - t + \log|t+1| \right) =$$

$$= 2 \left( \frac{x}{2} - \sqrt{x} + \log|\sqrt{x}+1| \right) + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

**15)**  $\int \frac{\cos x}{1+\sin^2 x} dx$  (ejercicio 581)

$$\int \frac{\cos x}{1+\sin^2 x} dx = \left[ \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \right] = \int \frac{1}{1+t^2} dt = \arctg t = \arctg(\sin x) + C$$

También se puede calcular como una integral inmediata:

$$\int \frac{\cos x}{1+\sin^2 x} dx = \int \frac{\overbrace{\cos x}^{\text{derivada}}}{1+\left(\underbrace{\sin x}_{\text{función}}\right)^2} dx = \arctg(\sin x) + C$$

**16)**  $\int \frac{x+2}{\sqrt{x+1}} dx$  (ejercicio 584)

$$\int \frac{x+2}{\sqrt{x+1}} dx = \left[ \begin{array}{l} t = x+1 \\ dt = dx \end{array} \right] = \int \frac{t+1}{\sqrt{t}} dt = \int \frac{t}{\sqrt{t}} dt + \int \frac{1}{\sqrt{t}} dt = \int t^{\frac{1}{2}} dt + \int t^{-\frac{1}{2}} dt = \frac{2}{3} \sqrt{t}^3 + 2\sqrt{t} =$$

$$= \frac{2}{3} \sqrt{(x+1)^3} + 2\sqrt{x+1} = \frac{2(x+1)\sqrt{x+1}}{3} + 2\sqrt{x+1} = \frac{(2x+4)\sqrt{x+1}}{3} + C$$

Con otro cambio de variable:

$$\int \frac{x+2}{\sqrt{x+1}} dx = \left[ \begin{array}{l} t^2 = x+1 \\ 2t dt = dx \end{array} \right] = \int \frac{t^2+1}{t} 2t dt = 2 \int (t^2+1) dt = 2t \left( \frac{t^3}{3} + t \right) = 2 \left( \frac{\sqrt{(x+1)^3}}{3} + \sqrt{x+1} \right) = \frac{(2x+4)\sqrt{x+1}}{3} + C$$

**17)**  $\int \frac{2x^3 - 9x^2 + 9x + 6}{x^2 - 5x + 6} dx$  (ejercicio 588)

Descomponemos en fracciones simples

$$\begin{array}{r} 2x^3 - 9x^2 + 9x + 6 \\ \hline x^2 - 5x + 6 \\ \hline -2x^3 + 10x^2 - 12x \\ \hline x^2 - 3x + 6 \\ \hline -x^2 + 5x - 6 \\ \hline 2x \end{array} \Rightarrow \frac{2x^3 - 9x^2 + 9x + 6}{x^2 - 5x + 6} = 2x + 1 + \frac{2x}{x^2 - 5x + 6}$$

$$\frac{2x}{x^2 - 5x + 6} = \frac{2x}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3) + B(x-2)}{(x-2)(x-3)} \Rightarrow$$

$$\Rightarrow 2x = A(x-3) + B(x-2) \Rightarrow \begin{cases} x=2 \Rightarrow A=-4 \\ x=3 \Rightarrow B=6 \end{cases}$$

$$\Rightarrow \frac{2x}{x^2 - 5x + 6} = \frac{-4}{x-2} + \frac{6}{x-3}$$

Integramos

$$\int \frac{2x^3 - 9x^2 + 9x + 6}{x^2 - 5x + 6} dx = \int (2x+1) dx + \int \frac{2x}{x^2 - 5x + 6} dx = \int 2x dx + \int 1 dx + \int \frac{-4}{x-2} dx + \int \frac{6}{x-3} dx = x^2 + x - 4 \log|x-2| + 6 \log|x-3| + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

**18)**  $\int_0^\pi e^x \sen x dx$  (ejercicio 588)

$$\int e^x \sen x dx = \left[ \begin{array}{l} u = e^x \xrightarrow{\text{derivamos}} du = e^x dx \\ dv = \sen x dx \xrightarrow{\text{integramos}} v = -\cos x \end{array} \right] = -e^x \cos x + \int e^x \cos x dx$$

Calculamos  $\int e^x \cos x dx$  por partes:

$$\int e^x \cos x dx = \left[ \begin{array}{l} u = e^x \xrightarrow{\text{derivamos}} du = e^x dx \\ dv = \cos x dx \xrightarrow{\text{integramos}} v = \sen x \end{array} \right] = e^x \sen x - \int e^x \sen x dx \text{ (donde la última integral es}$$

la misma que queremos calcular).

Así:

$$\int e^x \operatorname{sen} x dx = \left[ \begin{array}{l} u = e^x \xrightarrow{\text{derivamos}} du = e^x dx \\ dv = \operatorname{sen} x dx \xrightarrow{\text{integramos}} v = -\cos x \end{array} \right] = -e^x \cos x + \int e^x \cos x dx =$$

$$= -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x dx$$

Si llamamos  $I = \int e^x \operatorname{sen} x dx$ , se tiene que

$$I = -e^x \cos x + e^x \operatorname{sen} x - I \Rightarrow 2I = -e^x \cos x + e^x \operatorname{sen} x - e^x \cos x + e^x \operatorname{sen} x \Rightarrow$$

$$\Rightarrow I = \frac{-e^x \cos x + e^x \operatorname{sen} x}{2} = \frac{e^x (\operatorname{sen} x - \cos x)}{2}$$

y, por tanto,  $\int e^x \operatorname{sen} x dx = \frac{e^x (\operatorname{sen} x - \cos x)}{2} + C$

Llamamos  $G(x) = \frac{e^x (\operatorname{sen} x - \cos x)}{2} + C$  y aplicamos la regla de Barrow:

$$\int_0^\pi e^x \operatorname{sen} x dx = G(\pi) - G(0) = \frac{e^\pi + 1}{2}$$

**19)**  $\int_0^1 (x^2 + x) e^x dx$  (ejercicio 589)

$$\int (x^2 + x) e^x dx = \left[ \begin{array}{l} u = 2x + 1 \xrightarrow{\text{derivamos}} du = 2 dx \\ dv = e^x dx \xrightarrow{\text{integramos}} v = e^x \end{array} \right] = (2x + 1) e^x - 2 \int e^x dx =$$

$$= (2x + 1) e^x - 2e^x = (2x - 1) e^x + C = G(x)$$

Aplicando la regla de Barrow:

$$\int_0^1 (x^2 + x) e^x dx = G(\pi) - G(0) = e - 1$$

**20)**  $\int \frac{e^{2x} + e^x}{1 + e^{2x}} dx$  (ejercicio 589)

$$\int \frac{e^{2x} + e^x}{1 + e^{2x}} dx = \int \frac{(e^x + 1) e^x}{1 + e^{2x}} dx = \left[ \begin{array}{l} t = e^x \\ dt = e^x dx \end{array} \right] = \int \frac{t + 1}{1 + t^2} dt = \int \frac{t}{1 + t^2} dt + \int \frac{1}{1 + t^2} dx =$$

$$= \frac{1}{2} \log |1 + t^2| + \operatorname{arctg} t = \frac{1}{2} \log |1 + e^{2x}| + \operatorname{arctg}(e^x) + C$$

donde  $\log = \ln$  es el logaritmo natural o neperiano.

**21)**  $\int \operatorname{tg} x dx$  (ejercicio 590)

$$\int \operatorname{tg} x dx = \int \frac{\operatorname{sen} x}{\cos x} dx = -\log |\cos x| + C \text{ (inmediata)}$$

También se puede calcular por cambio de variable:

$$\int \operatorname{tg} x dx = \int \frac{\operatorname{sen} x}{\cos x} dx = \left[ \begin{array}{l} t = \cos x \\ dt = -\operatorname{sen} x dx \end{array} \right] = -\int \frac{1}{t} dt = -\log |t| = -\log |\cos x| + C$$

**22)**  $\int \frac{2}{1+\sqrt{x}} dx$  (ejercicio 591)

$$\int \frac{2}{1+\sqrt{x}} dx = \left[ \begin{array}{l} t^2 = x \\ 2t dt = dx \end{array} \right] = \int \frac{2}{1+t} 2t dt = 4 \int \frac{t}{1+t} dt$$

Descomponemos en fracciones simples  $\frac{t}{1+t}$  :

$$\frac{-t+1}{-1} \quad \frac{|t+1}{1} \quad \Rightarrow \quad \frac{t}{1+t} = 1 - \frac{1}{1+t}$$

Integramos

$$\int \frac{t}{1+t} dt = \int 1 dt - \int \frac{1}{1+t} dt = t - \log|1+t|$$

Así:

$$\int \frac{2}{1+\sqrt{x}} dx = \left[ \begin{array}{l} t^2 = x \\ 2t dt = dx \end{array} \right] = \int \frac{2}{1+t} 2t dt = 4 \int \frac{t}{1+t} dt = 4(t - \log|1+t|) = 4(\sqrt{x} - \log|1+\sqrt{x}|) + C$$

**23)**  $\int \frac{x+2}{x^2-2x+1} dx$  (ejercicio 594)

Descomponemos en fracciones simples:

$$\frac{x+2}{x^2-2x+1} = \frac{x+2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{A(x-1)+B}{(x-1)^2} \Rightarrow x+2 = A(x-1)+B$$

$$\Rightarrow \begin{cases} x=1 \Rightarrow B=3 \\ \text{Derivamos } x+2 = A(x-1)+B \\ 1=A \end{cases}$$

$$\Rightarrow \frac{x+2}{x^2-2x+1} = \frac{1}{x-1} + \frac{3}{(x-1)^2}$$

Integramos:

$$\int \frac{x+2}{x^2-2x+1} dx = \int \frac{1}{x-1} dx + \int \frac{3}{(x-1)^2} dx = \log|x-1| - \frac{3}{x-1} + C$$

**24)**  $\int_e^{e^3} \frac{\log x}{x} dx$  (ejercicio 597)

Calculamos una primitiva:

$$\int \frac{\log x}{x} dx = \left[ \begin{array}{l} t = \log x \\ dt = \frac{1}{x} dx \end{array} \right] = \int t dt = \frac{t^2}{2} = \frac{1}{2} \log^2 x + C = G(x)$$



(dicha primitiva, de hecho, es inmediata:  $\int \frac{\log x}{x} dx = \int \underbrace{\frac{1}{x}}_{\text{derivada}} \underbrace{\log x}_{\text{función}} dx = \frac{\log^2 x}{2} + C$ )

Aplicamos la regla de Barrow:

$$\int_e^{e^3} \frac{\log x}{x} dx = G(e^3) - G(e) = 4$$

**25)**  $\int_0^1 \frac{2 \operatorname{arctg} x}{1+x^2} dx$  (ejercicio 621)

Calculamos una primitiva:

$$\int \frac{2 \operatorname{arctg} x}{1+x^2} dx = 2 \int \frac{\operatorname{arctg} x}{1+x^2} dx = 2 \int \underbrace{\frac{1}{1+x^2}}_{\text{derivada}} \underbrace{\operatorname{arctg} x}_{\text{función}} dx = 2 \frac{\operatorname{arctg}^2 x}{2} = \operatorname{arctg}^2 x + C = G(x)$$

Aplicamos la regla de Barrow:

$$\int_0^1 \frac{2 \operatorname{arctg} x}{1+x^2} dx = G(1) - G(0) = \left(\frac{\pi}{4}\right)^2 - 0^2 = \frac{\pi^2}{16}$$

Directamente, por cambio de variable:

$$\int_0^1 \frac{2 \operatorname{arctg} x}{1+x^2} dx = \left[ \begin{array}{l} t = \operatorname{arctg} x \rightarrow \begin{cases} x = 1 \Rightarrow t = \frac{\pi}{4} \\ x = 0 \Rightarrow t = 0 \end{cases} \\ dt = \frac{1}{1+x^2} dx \end{array} \right] = 2 \int_0^{\frac{\pi}{4}} t dt = 2 \frac{t^2}{2} \Big|_0^{\frac{\pi}{4}} = t^2 \Big|_0^{\frac{\pi}{4}} = \operatorname{arctg}^2 x \Big|_0^1 = \left(\frac{\pi}{4}\right)^2 - 0^2 = \frac{\pi^2}{16}$$

**26)**  $\int [\cos(2x) + \operatorname{sen} x \cos x] dx$  (ejercicio 577)

$$\int [\cos(2x) + \operatorname{sen} x \cos x] dx = \int \cos(2x) dx + \int \operatorname{sen} x \cos x dx = \frac{1}{2} \operatorname{sen}(2x) + \frac{\operatorname{sen}^2 x}{2} + C$$

**27)**  $\int \frac{x^3 - 1}{x + 2} dx$  (ejercicio 577)

Descomponemos en fracciones simples:

$$\begin{array}{r} x^3 \quad + 1 \quad |x+2 \\ -x^3 - 2x^2 \\ \hline 2x^2 + 4x + 1 \\ -2x^2 + 4x \\ \hline 8x + 1 \\ -8x - 8 \\ \hline -7 \end{array} \Rightarrow \frac{x^3 + 1}{x + 2} = x^2 - 2x + 4 - \frac{7}{x + 2}$$

Integramos:

$$\int \frac{x^3+1}{x+2} dx = \int (x^2 - 2x + 4) dx - 7 \int \frac{1}{x+2} dx = \frac{x^3}{3} - x^2 + 4x - 7 \log|x+2| + C$$

**28)**  $\int \frac{3x}{x^2+2x+3} dx$  (ejercicio 605)

El polinomio  $x^2+2x+3$  no tiene raíces reales. Ahora bien:

$$\begin{aligned} \int \frac{3x}{x^2+2x+3} dx &= \frac{1}{2} \int \frac{2 \cdot 3x}{x^2+2x+3} dx = \frac{1}{2} \int \frac{6x}{x^2+2x+3} dx = \frac{1}{2} \int \frac{6x+6-6}{x^2+2x+3} dx = \\ &= \frac{1}{2} \int \frac{6x+6}{x^2+2x+3} dx + \frac{1}{2} \int \frac{-6}{x^2+2x+3} dx = \frac{1}{2} \underbrace{\int \frac{3 \left( \begin{smallmatrix} \text{derivada del denominador} \\ 2x+2 \end{smallmatrix} \right)}{x^2+2x+3} dx}_{\text{inmediata}} + \frac{1}{2} \int \frac{-6}{x^2+2x+3} dx \end{aligned}$$

Calculamos  $\int \frac{-6}{x^2+2x+3} dx$ :

$$\begin{aligned} \frac{1}{2} \int \frac{-6}{x^2+2x+3} dx &\stackrel{(1)}{=} -3 \int \frac{1}{(x+1)^2+2} dx = -3 \int \frac{\frac{1}{2}}{\frac{(x+1)^2+2}{2}} dx = -\frac{3}{2} \int \frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1} dx = \\ &= -\frac{3}{2} \sqrt{2} \int \frac{\frac{1}{\sqrt{2}}}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1} dx = -\frac{3\sqrt{2}}{2} \operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) \end{aligned}$$

Donde en (1) hemos tenido en cuenta que  $(x+1)^2+2 = x^2+1+2x+2 = x^2+2x+3$

Así:

$$\int \frac{3x}{x^2+2x+3} dx = \frac{3}{2} \log|x^2+2x+3| - \frac{3\sqrt{2}}{2} \operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) + C$$