TEMA 12 – CÁLCULO DE PRIMITIVAS

EJERCICIO : Calcula las integrales:

a) $\int \frac{4x-2\cos x}{x^2-\text{sen}\,x}\,dx$ **b)** $\int x\,e^{2x}\,dx$ **c)** $\int \frac{x}{x^2-9}\,dx$ **d)** $\int \frac{e^x}{1+e^{2x}}\,dx$

e) $\int \frac{e^x}{1+(e^x)^2}\,dx$ **f)** $\int \frac{dx}{x^3+2x^2+x}$ **g)** $\int \frac{dx}{\sqrt[4]{3x+1}}$ **h)** $\int (2x+1)\cos x\,dx$

i) $\int \frac{2}{x^2-1}\,dx$ **j)** $\int 2x(x-1)^2\,dx$ **k)** $\int (x+2)\,\text{sen}\,x\,dx$ **l)** $\int \frac{dx}{x^3-3x^2}$

m) $\int \frac{3x+\sqrt{x}}{x^2}\,dx$ **n)** $\int (x+3)\,e^{x+1}\,dx$ **ñ)** $\int \frac{x}{x^2+2x-8}\,dx$

Solución:

a) $\int \frac{4x-2\cos x}{x^2-\text{sen}\,x}\,dx = \int \frac{2\,(2x-\cos x)}{x^2-\text{sen}\,x}\,dx = 2\ln\left|x^2-\text{sen}\,x\right|+k$

b) $\int x\,e^{2x}\,dx$. Integramos por partes: $\begin{cases} u = x \;\rightarrow\; du = dx \\ dv = e^{2x}\,dx \;\rightarrow\; v = \frac{1}{2}e^{2x} \end{cases}$

$$\int x\,e^{2x}\,dx = \frac{x}{2}\,e^{2x} - \int \frac{1}{2}\,e^{2x}\,dx = \frac{x}{2}\,e^{2x} - \frac{1}{4}\,e^{2x} + k = \left(\frac{x}{2}-\frac{1}{4}\right)e^{2x}+k$$

c) $\int \frac{x}{x^2-9}\,dx$. Descomponemos en fracciones simples:

$$\frac{x}{x^2-9} = \frac{x}{(x-3)\,(x+3)} = \frac{A}{x-3}+\frac{B}{x+3} = \frac{A\,(x+3)+B\,(x-3)}{(x-3)\,(x+3)}$$

- Para $x=3 \;\rightarrow\; 3=6A \;\rightarrow\; A=\frac{1}{2}$

- Para $x=-3 \;\rightarrow\; -3=-6B \;\rightarrow\; B=\frac{1}{2}$

Por tanto: $\int \frac{x}{x^2-9}\,dx = \int \left(\frac{\frac{1}{2}}{x-3}+\frac{\frac{1}{2}}{x+3}\right)dx = \frac{1}{2}\ln|x-3|+\frac{1}{2}\ln|x+3|+k$

d) $\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx = \text{arctg}\left(e^x\right)+k$

e) $\int (x+2)\ln x\,dx$. Integramos por partes: $\begin{cases} u = \ln x \;\rightarrow\; du = \frac{1}{x}\,dx \\ dv = (x+2)\,dx \;\rightarrow\; v = \frac{x^2}{2}+2x \end{cases}$

$$\int (x+2)\ln x\,dx = \left(\frac{x^2}{2}+2x\right)\ln x - \int\left(\frac{x^2}{2}+2x\right)\frac{1}{x}\,dx = \left(\frac{x^2}{2}+2x\right)\ln x - \int\left(\frac{x}{2}+2\right)dx =$$

$$= \left(\frac{x^2}{2}+2x\right)\ln x - \frac{x^2}{4} - 2x + k$$

f) $\int \frac{dx}{x^3+2x^2+x}$. Descomponemos en fracciones simples:

$$\frac{1}{x^3+2x^2+x}=\frac{1}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}=\frac{A(x+1)^2+Bx(x+1)+Cx}{x(x+1)^2}$$

- Para $x=0 \rightarrow 1=A$
- Para $x=-1 \rightarrow 1=-C \rightarrow C=-1$
- Para $x=1 \rightarrow 1=4A+2B+C \rightarrow B=-1$

Por tanto: $\int \frac{dx}{x^3+2x^2+x}=\int\left(\frac{1}{x}+\frac{-1}{x+1}+\frac{-1}{(x+1)^2}\right)dx=\ln|x|-\ln|x+1|+\frac{1}{x+1}+k$

g) $\int \frac{dx}{\sqrt[4]{3x+1}}=\int (3x+1)^{-1/4}\,dx=\frac{1}{3}\int 3\,(3x+1)^{-1/4}\,dx=\frac{1}{3}\,\frac{(3x+1)^{3/4}}{3/4}+k==\frac{4\sqrt[4]{(3x+1)^3}}{9}+k$

h) $\int (2x+1)\cos x\,dx$. Integramos por partes: $\begin{cases} u=2x+1 \rightarrow du=2dx \\ dv=\cos x\,dx \rightarrow v=\operatorname{sen} x\end{cases}$

$$\int (2x+1)\cos x\,dx=(2x+1)\operatorname{sen} x-\int 2\operatorname{sen} x\,dx=(2x+1)\operatorname{sen} x+2\cos x+k$$

i) $\int \frac{2}{x^2-1}\,dx$. Descomponemos en fracciones simples:

$$\frac{2}{x^2-1}=\frac{2}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}=\frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$$

- Para $x=1 \rightarrow 2=2A \rightarrow A=1$
- Para $x=-1 \rightarrow 2=-2B \rightarrow B=-1$

Por tanto: $\int \frac{2}{x^2-1}\,dx=\int\left(\frac{1}{x-1}+\frac{-1}{x+1}\right)dx=\ln|x-1|-\ln|x+1|+k$

j) $\int 2x(x-1)^2\,dx=\int 2x\left(x^2-2x+1\right)dx=\int\left(2x^3-4x^2+2x\right)dx=\frac{2x^4}{4}-\frac{4x^3}{3}+x^2+k==\frac{x^4}{2}-\frac{4x^3}{3}+x^2+k$

k) $\int (x+2)\operatorname{sen} x\,dx$. Integramos por partes: $\begin{cases} u=x+2 \rightarrow du=dx \\ dv=\operatorname{sen} x\,dx \rightarrow v=-\cos x\end{cases}$

$$\int (x+2)\operatorname{sen} x\,dx=-(x+2)\cos x+\int \cos x\,dx=-(x+2)\cos x+\operatorname{sen} x+k$$

l) $\int \frac{dx}{x^3-3x^2}$. Descomponemos en fracciones simples:

$$\frac{1}{x^3-3x^2}=\frac{1}{x^2(x-3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-3}=\frac{Ax(x-3)+B(x-3)+Cx^2}{x^2(x-3)}$$

- Para $x=0 \rightarrow 1=-3B \rightarrow B=\frac{-1}{3}$
- Para $x=3 \rightarrow 1=9C \rightarrow C=\frac{1}{9}$
- Para $x=1 \rightarrow 1=-2A-2B+C \rightarrow A=\frac{-1}{9}$

Por tanto: $\int \frac{dx}{x^3-3x^2}=\int \frac{\frac{-1}{9}}{x}\,dx+\int \frac{\frac{-1}{3}}{x^2}\,dx+\int \frac{\frac{1}{9}}{x-3}\,dx=\frac{-1}{9}\ln|x|+\frac{1}{3x}+\frac{1}{9}\ln|x-3|+k$

m) $\int \frac{3x+\sqrt{x}}{x^2}\,dx=\int\left(\frac{3x}{x^2}+\frac{\sqrt{x}}{x^2}\right)dx=\int\left(\frac{3}{x}+x^{-3/2}\right)dx=3\ln|x|+\frac{x^{-1/2}}{\frac{-1}{2}}+k==3\ln|x|-\frac{2}{\sqrt{x}}+k$

n) $\int (x+3)\,e^{x+1}\,dx$. Integramos por partes: $\begin{cases} u=x+3 \rightarrow du=dx \\ dv=e^{x+1}dx \rightarrow v=e^{x+1}\end{cases}$

$$\int (x+3)\,e^{x+1}dx=(x+3)\,e^{x+1}-\int e^{x+1}dx=(x+3)\,e^{x+1}-e^{x+1}+k=e^{x+1}(x+3-1)+k==(x+2)\,e^{x+1}+k$$

ñ) $\int \frac{x}{x^2+2x-8}\,dx$. Descomponemos en fracciones simples:

$$\frac{x}{x^2+2x-8}=\frac{x}{(x+4)\,(x-2)}=\frac{A}{x+4}+\frac{B}{x-2}=\frac{A(x-2)+B(x+4)}{(x+4)\,(x-2)}$$

- Para $x=2 \;\rightarrow\; 2=6B \;\rightarrow\; B=\frac{1}{3}$

- Para $x=-4 \;\rightarrow\; -4=-6A \;\rightarrow\; A=\frac{2}{3}$

Por tanto: $\int \frac{x}{x^2+2x-8}\,dx=\int\left(\frac{\frac{2}{3}}{x+4}+\frac{\frac{1}{3}}{x-2}\right)dx=\frac{2}{3}\ln\left|x+4\right|+\frac{1}{3}\ln\left|x-2\right|+k$